Name ______________________ Instructor ______________________

Technical Calculus (I)
Math 210 - Test 2
October 19, 1998

You must show all relevant work to receive full credit. The point value of each problem is shown in the left hand margin.

(10) 1. Find $f(a)$ and $f(2a)$, if $f(x) = ax^2 - a^2x$.

(10) 2. A certain object, after being heated, cools at such a rate that its temperature T (in degrees Celsius) decreases 10% each minute. If the object is originally heated to 100^oC find $\lim_{t \to 10} T$ and $\lim_{t \to \infty} T$, where t is the time (in minutes).

(10) 3. Find the average rate of change of $y = 1 - 2x^2$ with respect to x from $P(1, -1)$ to $Q(1.1, -1.42)$. Then compare this with the instantaneous rate of change of y with respect to x at P by finding m_{tan} at P.

(10) 4. Find the derivative of $y = x^2 - \frac{2}{x}$ by using the *delta-process.* Then evaluate the derivative at the point $(-2, 5)$.

(10) 5. The value in dollars of a certain car is given by the function $V = \frac{6000}{t+1}$, where t is measured in years. Find a general expression for the rate of change of V with respect to t, and then evaluate this expression when $t = 3$ years.

(10) 6. Find the slope of the tangent line to the curve $y = 2x^6 - 6x^2$ at the point with abscissa $x = 2$.

(10) 7. In thermodynamics, an equation relating pressure p, volume V and temperature T is

$$(p + \frac{a}{V^2})(V - b) = RT$$

where a, b and R are constants. This equation is known as the *van der Waalls* equation. Solve this equation for p, and then find the expression for the rate of change of p with respect to V, assuming a constant temperature.

(10) 8. Evaluate the derivative of $y = x^2 \sqrt[3]{3x + 2}$ at $x = 1$.

(10) 9. Find $\frac{dy}{dx}$ by implicit differentiation.

$$xy^3 + 3y + x^2 = 9$$

(10) 10. Find the second derivative of $y = \sqrt[3]{6x + 5}$.