Math 160 - Fall 2004
G. C. Neumann
11:30 MWF

Midterm #2
October 18, 2004

Name: _____

PLEASE READ THIS PAGE!!!

1. **Hints:**
   - You might want to quickly look over all of the questions and start by working the questions that are easiest for you.
   - Check your answers only if you have time.
   - Many of the questions have multiple parts. Don't automatically give up on a question because you don't know how to do one part.
   - Consider showing your work, even when it's not requested. This could help you earn partial credit for an incorrect answer.
   - You don't get extra points for finishing early. If you have extra time, please consider checking your work over one more time before turning in your paper.
2. **Reminders:**
   - You may use a calculator, provided that your calculator cannot be used to communicate with others (for example, no cell phones are allowed).
   - You are allowed to use the sheet of notes that you prepared for use with this exam (both sides of one $4'' \times 6''$ note card with handwritten notes). These notes must be turned in with your exam. No other type of written materials is allowed.
3. Read the following statement and sign your name:

   On my honor, as a student, I have neither given nor received unauthorized aid on this academic work.

   Signature: _____
4. Please make sure that your exam contains *seven pages*, including this one.

Question	Score	Possible	Question	Score	Possible
1		10	5		35
2		10	6		15
3		5	7		10
4		15			
			Total		100

1. (2 points each) Circle either "True" or "False" for each of the following:

   a) True False: Arrow's theorem implies that there is no voting system for three or more alternatives (other than a dictatorship) that satisfies both the Condorcet winner criterion and independence of irrelevant alternatives.

   b) True False: Consider an election with an odd number of voters and two candidates. Majority rule is the only voting system that satisfies anonymity (all voters treated the same), neutrality (switching the order of the alternatives switches the result), and monotonicity.

   c) True False: If a voting system satisfies monotonicity, then a winning alternative will still be a winner if a voter improves the ranking of this winning alternative on his ballot.

   d) True False: Consider the weighted voting system [q: w(A), w(B), w(C)] = [5: 3, 2, 1]. Then B and C are both critical voters in the blocking coalition {B, C}.

   e) True False: Consider the weighted voting system [q: w(A), w(B), w(C)] = [5: 3, 2, 1]. Then A and B each have veto power.

2. (5 points each) Show your work in an easy to follow manner. Suppose that you have just been dealt a hand of five cards. No two cards are alike. Calculate the following:

   (a) How many different ways can you arrange these five cards?

   (b) You want to keep three of the five cards. How many different hands of three cards are possible from your original hand of five cards? The order of the cards doesn't matter here.

3. (5 points) Are the following two weighted voting systems equivalent? Explain.

System 1: [q: w(A), w(B), w(C), w(D)] = [5: 3, 1, 1, 1]
System 2: [q: w(A), w(B), w(C), w(D)] = [8: 5, 2, 2, 1]

4. (5 points each) The following questions ask you to describe an election with four alternatives and four voters that illustrates the Condorcet voting paradox.

   (a) Construct a table containing a set of preference lists that illustrates this situation.

   (b) Write a story problem describing the four voter situation above.

   (c) Explain why this situation seems paradoxical.

5. (5 points each) Consider the following set of preference lists for a four candidate, 17 voter election:

	Number of voters (17)			
Rank	7	5	4	1
First	A	B	C	B
Second	D	A	B	C
Third	C	D	A	A
Fourth	B	C	D	D

For your convenience, this table will be repeated on the next two pages.

   (a) Find the winner(s) under plurality voting. Show your work.

	Number of voters (17)			
Rank	7	5	4	1
First	A	B	C	B
Second	D	A	B	C
Third	C	D	A	A
Fourth	B	C	D	D

(b) Find the winner(s) under Borda count. Show your work.

(c) Find the winner(s) under sequential pairwise voting with the agenda B, A, C, D. Show your work.

(d) Find the winner(s) under the Hare system. Show your work.

	Number of voters (17)			
Rank	7	5	4	1
First	A	B	C	B
Second	D	A	B	C
Third	C	D	A	A
Fourth	B	C	D	D

(e) Find all pairs of alternatives X and Y where all of the voters prefer X to Y. Do all of the voting systems listed above satisfy the Pareto condition for this particular election? Explain.

(f) Do all of the voting systems listed above satisfy the Condorcet winner criterion for this particular election? Explain.

(g) Find the winners under approval voting if the top two candidates are to be elected. Assume that a voter approves of a candidate if that candidate is in first or second place on that voter's preference list.
Suggestion: Write out the resulting ballot table for approval voting before doing your calculation.

6. Consider the following weighted voting system:

$$[q\text{: } w(A), w(B), w(C), w(D)] = [5\text{: } 3, 1, 1, 1]$$

(a) (6 points) Calculate the Banzhaf index for this voting system. Show your work.

(b) (6 points) Calculate the Shapley-Shubik index for this voting system. Show your work.

(c) (3 points) Does A have the same relative power in both models? Show your work.

7. An elementary school has a personnel committee consisting of the principal (P) and four teachers (A, B, C, D). The committee approves a motion if a simple majority of the four teachers supports it. If the four teachers are deadlocked, the principal's vote is the deciding vote.

   (a) (5 points) List the minimal winning coalitions for this committee.

   (b) (5 points) Is power shared equally by the members of this committee? Explain.