Math 160 - Fall 2004
G. C. Neumann
11:30 MWF

Midterm #1

September 17, 2004

Name: _____

PLEASE READ THIS PAGE!!!

1. **Hints:**
   - You might want to quickly look over all of the questions and start by working the questions that are easiest for you.
   - Check your answers only if you have time.
   - Many of the questions have multiple parts. Don't automatically give up on a question because you don't know how to do one part.
   - Consider showing your work, even when it's not requested. This could help you earn partial credit for an incorrect answer.
   - You don't get extra points for finishing early. If you have extra time, please consider checking your work over one more time before turning in your paper.
2. **Reminders:**
   - You may use a calculator, provided that your calculator cannot be used to communicate with others (for example, no cell phones are allowed).
   - You are allowed to use the sheet of notes that you prepared for use with this exam (both sides of one $4'' \times 6''$ note card with handwritten notes). These notes must be turned in with your exam. No other type of written materials is allowed.
3. Read the following statement and sign your name:

   On my honor, as a student, I have neither given nor received unauthorized aid on this academic work.

   Signature: _____
4. Please make sure that your exam contains *seven pages*, including this one.

Question	Score	Possible	Question	Score	Possible
1		16	5		10
2		15	6		5
3		27	7		5
4		12	8		10
			Total		100

1. (2 points each) Circle either "True" or "False" for each of the following:

a) True False: To check that you have found a spanning tree for a graph, it is enough to check that each vertex is an endpoint of some edge in your collection of edges.

b) True False: When you Eulerize a street network graph, the added edges correspond to building new roads.

c) True False: If every vertex in a connected graph has even valence, then it is always possible to find a path that uses every edge exactly once and that takes you back to your starting vertex.

d) True False: In a complete graph, it is always possible to find a path that visits every vertex exactly once and that takes you back to your starting vertex.

e) True False: In contrast to the sorted edges algorithm for TSP, Kruskal's algorithm is guaranteed to find a minimum cost spanning tree in every connected graph.

f) True False: If a connected graph has exactly two odd vertices, then it is possible to find a path that uses every edge exactly once. However, the starting vertex and the ending vertex will be different.

g) True False: The Euler circuits in a graph always have the same cost.

h) True False: The Hamiltonian circuits in a graph always have the same cost.

2. (5 points each) Calculate the following. Show your work in an easy to follow manner.

(a) How many different Hamiltonian circuits are possible in a complete graph with 8 vertices?

(b) You have three different K-State baseball caps, five different K-State tee shirts, and two different K-State jackets. How many different K-State outfits can you put together for a cool autumn day?

(c) You are in charge of a school raffle. How many different raffle tickets can go on sale if the ticket number consists of two upper case letters A - Z, followed by two single digit numbers $0 - 9$. The single digit numbers can be the same, except that "00" is not allowed. The letters must be different.

3. We have been studying graphs and some particular problem types that use graphs. For each of the situations below, please answer the following:

   - (2 points) What would a vertex represent? If there is a number associated with a vertex, what does it represent?
   - (2 points) What would an edge represent? If there is a number associated with an edge, what does it represent?
   - (5 points) Which type of graph problem would best fit the situation? The problem could be an Euler circuit problem, a Chinese postman problem (Eulerization), a Hamiltonian circuit problem, a traveling salesman problem, a minimum cost spanning tree problem, or a critical path problem. Explain why your choice makes sense.

   (a) You are working as a textbook sales representative. Your service territory consists of Kansas State, the University of Kansas, Emporia State, Hays State, Wichita State, and Hutchinson Community College. You know the travel times between every pair of schools and want to know the most efficient order for visiting all of the schools in your service territory exactly once each.

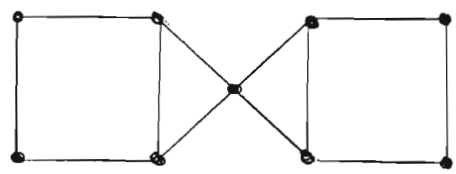
(b) You are setting up a weekday shuttle bus service between Kansas State, the University of Kansas, Emporia State, Hays State, Wichita State, and Hutchinson Community College. You know how much it will cost to run a bus between every pair of schools. You want to design your shuttle bus service so that it is possible to get from one school to another. However, you want to keep your overall cost as low as possible.

(c) You are working for Kansas State University and have been asked to plan a route for inspecting all of the roads on the main campus for potholes. Your crew will be staying off the streets that border the campus (such as Dennison and Anderson). You were told to assume that all of the campus roads are two-way streets.

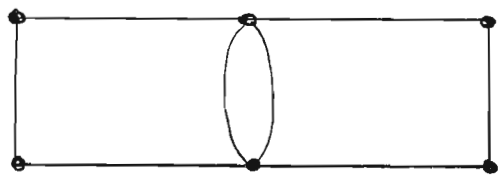
4. For each of the following street networks, answer the following:

- (2 points) Does an Euler circuit exist? Explain.
- (4 points) If an Euler circuit exists, show an Euler circuit.
- (4 points) If an Euler circuit doesn't exist, find an Eulerization. Is it possible to find an Eulerization that reuses fewer edges? Explain.

(a)

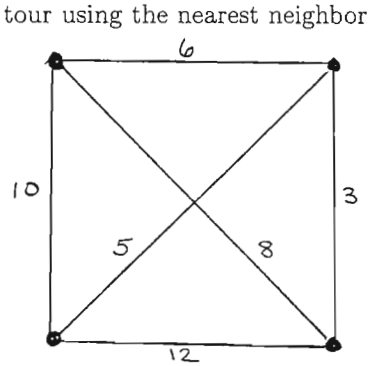


(b)

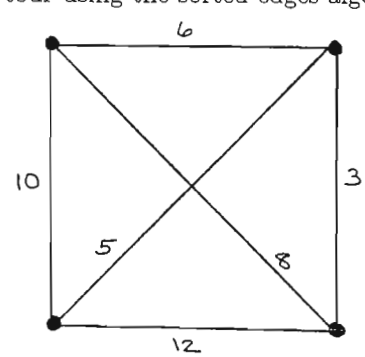


5. (5 points each) Find the following. Do you need to choose a starting vertex? If so, please specify your starting vertex.

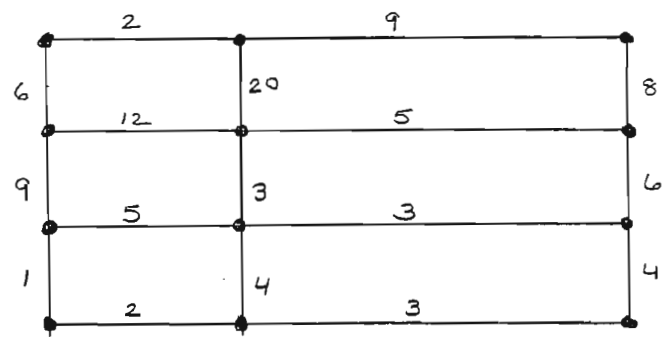
(a) Find a TSP tour using the nearest neighbor algorithm for the following graph:



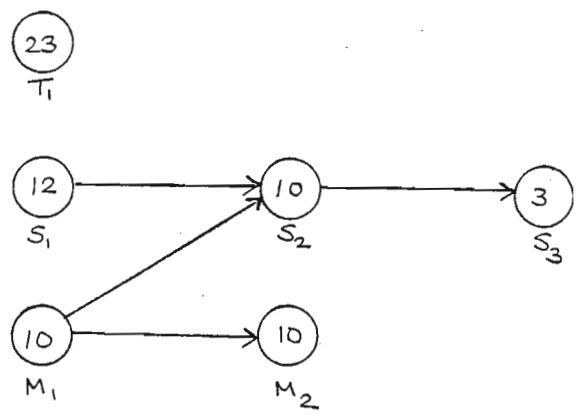
(b) Find a TSP tour using the sorted edges algorithm for the following graph:



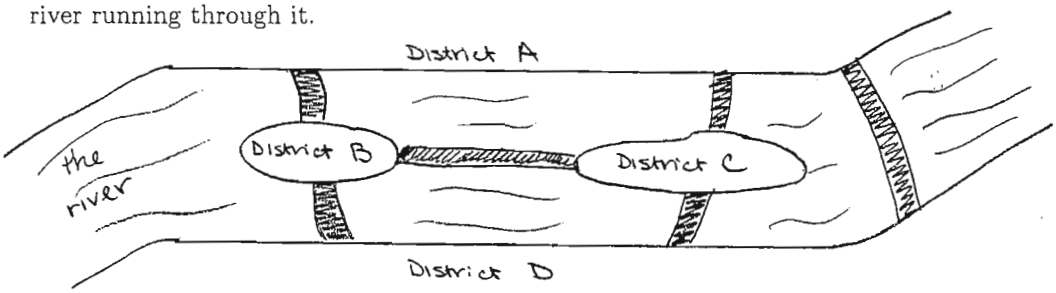
6. (5 points) Find a minimum cost spanning tree for the following graph.



7. (5 points) Consider the following order requirement digraph. All times are given in hours. What is the earliest completion time for this project? Show your work.



8. The figure below shows a map of a city (not Königsberg) with four districts, six bridges, and a river running through it.



(a) (8 points) Suppose that Hurricane Genevra has destroyed all of the bridges. The city needs to build enough bridges so that it is possible to travel between any pair of districts by car. The city has the following estimates for building bridges:

Districts	Cost (in millions of dollars)
AB	1
AC	1
AD	20
BC	5
BD	10
CD	10

What is the lowest cost for building bridges so that it is possible to travel between any pair of districts by car? Show your work.

(b) (2 points) Would anyone be inconvenienced by the new bridge layout? Explain.