1. (2 points each) Circle either "True" or "False" for each of the following:

a) True **False** To check that you have found a spanning tree for a graph, it is enough to check that each vertex is an endpoint of some edge in your collection of edges.

b) True **False** When you Eulerize a street network graph, the added edges correspond to building new roads.

c) **True** False: If every vertex in a connected graph has even valence, then it is always possible to find a path that uses every edge exactly once and that takes you back to your starting vertex.

d) **True** False: In a complete graph, it is always possible to find a path that visits every vertex exactly once and that takes you back to your starting vertex.

e) **True** False: In contrast to the sorted edges algorithm for TSP, Kruskal's algorithm is guaranteed to find a minimum cost spanning tree in every connected graph.

f) **True** False: If a connected graph has exactly two odd vertices, then it is possible to find a path that uses every edge exactly once. However, the starting vertex and the ending vertex will be different.

g) **True** False: The Euler circuits in a graph always have the same cost.

h) True **False** The Hamiltonian circuits in a graph always have the same cost.

2. (5 points each) Calculate the following. Show your work in an easy to follow manner.

(a) How many different Hamiltonian circuits are possible in a complete graph with 8 vertices?

H-ckts for complete graph with n vert is $\frac{(n-1)!}{2}$

$$\# \text{ H-ckts} = \frac{(8-1)!}{2} = \frac{7!}{2} = \frac{7\times6\times5\times4\times3\times2\times1}{2} = 2520$$

(b) You have three different K-State baseball caps, five different K-State tee shirts, and two different K-State jackets. How many different K-State outfits can you put together for a cool autumn day?

outfit :

#choices	3	5	2

different outfits $= 3\times5\times2 = 30$

(c) You are in charge of a school raffle. How many different raffle tickets can go on sale if the ticket number consists of two upper case letters A - Z, followed by two single digit numbers 0 – 9? The single digit numbers can be the same, except that "00" is not allowed. The letters must be different.

A-Z A-Z 0-9 0-9

different can't be 00

There are $26 \times 25 \times 10 \times 10$ possibilities for A-Z A-Z 0-9 0-9 $= 65{,}000$

letters different

and $26 \times 25 \times 1 \times 1$ possibilities for A-Z A-Z 0 0 $= 650$

different

different raffle tickets $= 65{,}000 - 650 = 64{,}350$

2 digit #, not zero (99 choices)

Alt A-Z A-Z

different

different raffle tickets $= 26 \times 25 \times 99 = 64{,}350$

3. We have been studying graphs and some particular problem types that use graphs. For each of the situations below, please answer the following:

- (2 points) What would a vertex represent? If there is a number associated with a vertex, what does it represent?
- (2 points) What would an edge represent? If there is a number associated with an edge, what does it represent?
- (5 points) Which type of graph problem would best fit the situation? The problem could be an Euler circuit problem, a Chinese postman problem (Eulerization), a Hamiltonian circuit problem, a traveling salesman problem, a minimum cost spanning tree problem, or a critical path problem. Explain why your choice makes sense.

(a) You are working as a textbook sales representative. Your service territory consists of Kansas State, the University of Kansas, Emporia State, Hays State, Wichita State, and Hutchinson Community College. You know the travel times between every pair of schools and want to know the most efficient order for visiting all of the schools in your service territory exactly once each.

Vertex — school (K-State, etc)

Edge — route between a pair of schools. Weight = travel time

Visiting each school (vertex) once — assume end up at start — is a H-ckt. But we want most efficient H-ckt and we have an edge for every pair of vert (ie, have a complete graph), so TSP best fits this situation.

Note 1) Travelling all edges not the issue here (not Euler ckt, not Eulerization).

2) Min cost spanning tree lets you reach all of the vert, but it can visit a vert more than once and might not give the lowest cost for visiting each vert on a path (just gives lowest cost graph that makes it possible to visit all vert).

3) Critical path problems are for finding the earliest completion time for a project with multiple tasks (some dependent on others, some can run at same time).

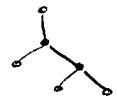
(b) You are setting up a weekday shuttle bus service between Kansas State, the University of Kansas, Emporia State, Hays State, Wichita State, and Hutchinson Community College. You know how much it will cost to run a bus between every pair of schools. You want to design your shuttle bus service so that it is possible to get from one school to another. However, you want to keep your overall cost as low as possible.

Vertex – school (K-State, etc)

Edge – route between 2 schools Weight = cost of running bus on route.

Minimum cost spanning tree – Have an edge between every pair of schools, so complete graph (not relevant here) and connected. Want to get between any pair of schools by bus (spanning tree) with the overall cost minimized.

<u>Note</u>: <u>Not</u> TSP. It doesn't matter if every vertex is visited exactly once and a circuit isn't required.



gets between every pair of schools.

(c) You are working for Kansas State University and have been asked to plan a route for inspecting all of the roads on the main campus for potholes. Your crew will be staying off the streets that border the campus (such as Dennison and Anderson). You were told to assume that all of the campus roads are two-way streets.

Street network – to inspect streets, need to travel on each street.

Vertex – Street intersections

Edge – street.

Want to travel each edge once. Assume that want to end up at starting location. Would like to find an E-ckt

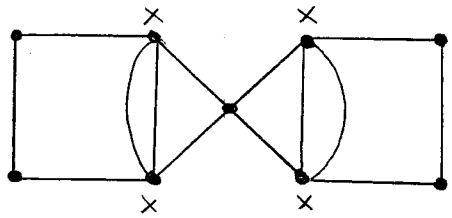
* If you need to travel both sides of the street to do the inspection, then every vertex is even and an E-ckt exists. (an edge for each street direction; i.e., 2 edges for each street)

* If you can do the inspection by travelling one direction on the street, have one edge for each street and some vertices are odd (for example, at Dennison and Claflin). So no E-ckt and have to Eulerize (Chinese postman problem.)

4. For each of the following street networks, answer the following:

- (2 points) Does an Euler circuit exist? Explain.
- (4 points) If an Euler circuit exists, show an Euler circuit.
- (4 points) If an Euler circuit doesn't exist, find an Eulerization. Is it possible to find an Eulerization that reuses fewer edges? Explain.

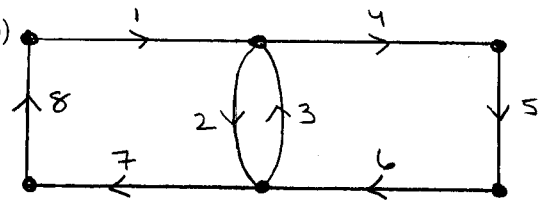
(a)



Have four odd vert, so Euler's thm tells us that there is no E-ckt.

Can pair the odd vert by reusing an existing edges, as in the Eulerization given. Reusing $\frac{\#\text{odd}}{2} = \frac{4}{2} = 2$ edges is the best possible Eulerization and can't reuse fewer edges.

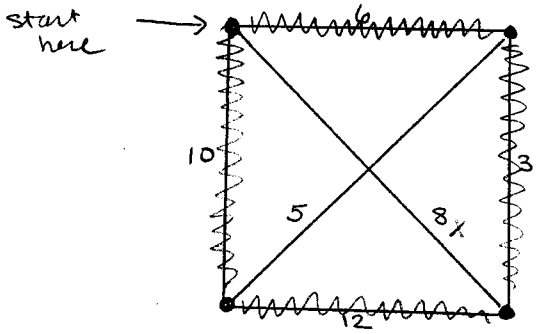
(b)



Other answers are possible!

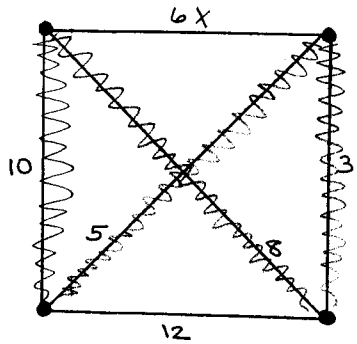
5. (5 points each) Find the following. Do you need to choose a starting vertex? If so, please specify your starting vertex.

(a) Find a TSP tour using the nearest neighbor algorithm for the following graph:



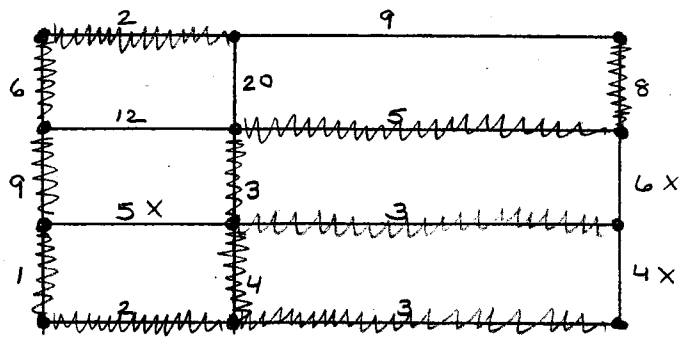
In general, Nearest neighbor depends on your choice of starting vertex

(b) Find a TSP tour using the sorted edges algorithm for the following graph:



Sorted edges does not depend on starting vertex

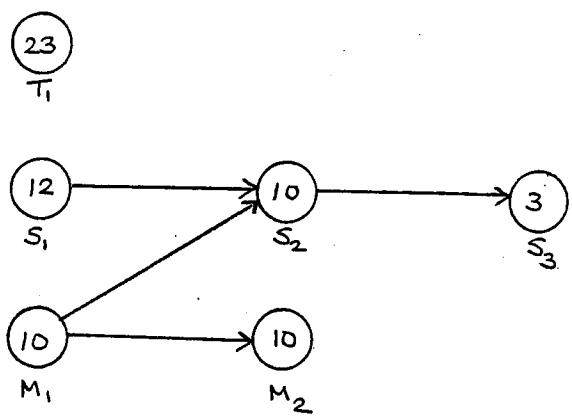
6. (5 points) Find a minimum cost spanning tree for the following graph.



Other answers are possible (depending on which edge costing 4 you chose first and which edge costing 9 you choose)

(In case you're curious, the minimum cost is 46)

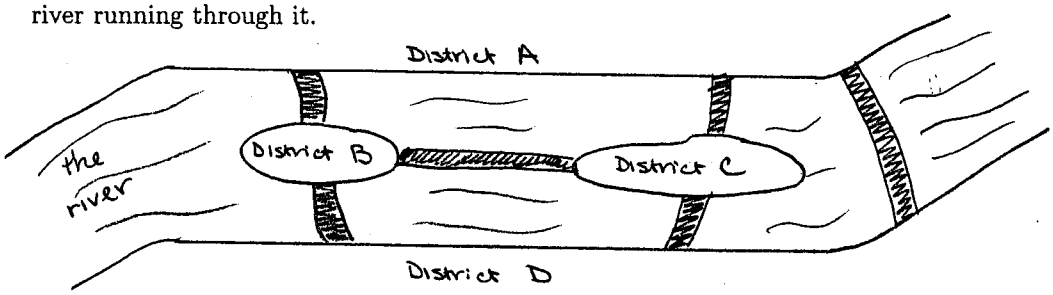
7. (5 points) Consider the following order requirement digraph. All times are given in hours. What is the earliest completion time for this project? Show your work.



	cost (hours)
T_1	23
$S_1 S_2 S_3$	$12+10+3 = 25$
$M_1 S_2 S_3$	$10+10+3 = 23$
$M_1 M_2$	$10+10 = 20$

Earliest completion time is <u>25 hours</u>

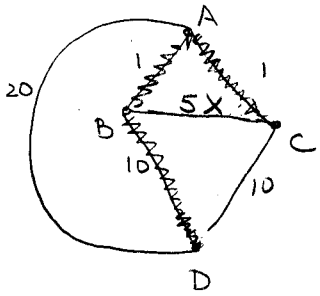
8. The figure below shows a map of a city (not Königsberg) with four districts, six bridges, and a river running through it.



(a) (8 points) Suppose that Hurricane Genevra has destroyed all of the bridges. The city needs to build enough bridges so that it is possible to travel between any pair of districts by car. The city has the following estimates for building bridges:

Districts	Cost (in millions of dollars)
AB	1
AC	1
AD	20
BC	5
BD	10
CD	10

What is the lowest cost for building bridges so that it is possible to travel between any pair of districts by car? Show your work.



Want min cost spanning tree.

Apply Kruskal's algorithm (guaranteed to find min cost, though it can give different spanning trees when there are edges with the same cost)

Cost = 1+1+10 million dollars = $12 million.

(b) (2 points) Would anyone be inconvenienced by the new bridge layout? Explain.

Yes! For example, to get from B to C, have to go from B to A and then from A to C

Another example, to get from C to D, have to go from C to A, then A to B and finally B to D.