# MATHEMATICS, IT'S FORM AND IMPACT
## Section A Hour Exam 2
## November 1994

In all multiple choice problems **circle** the correct answer!

Name _____

**(4 pts)** 1. Give an example of

(a) a dimension _____

(b) a unit of length _____

(c) a unit of money _____

(d) a unit of pressure _____

**(4 pts)** 2. Solve the following dimension equation

$$mi = d \cdot \text{sec} \qquad d = _____$$

**(4 pts)** 3. If there are 12 inches in 1 foot and 3 feet in a yard, use the cancellation techniques to show how many cubic inches are in 3 cubic yards?

$$3\text{yd}^3 \cdot \frac{(\quad)}{(\quad)} \cdot$$

$$= _____ \text{ in}^3$$

**(4 pts)** 4. Use dimensions to set up and solve the problem. How many hours will it take a truck traveling 100 feet per second to travel 160 miles? [1 mile = 5280 feet.]

**(4 pts)** 5. Arrow's Impossibility Theorem implies

A) that all preferential voting methods are equally bad.

B) that in every election, no matter what preferential voting method we use, at least one of the fairness criteria will be violated.

C) that it is impossible to rule out the possibility that some day, someone may discover a preferential voting method that doesn't violate any of the fairness criteria.

D) that it is impossible to have a preferential voting method that satisfies all of the fairness criteria.

E) None of the above.

Questions 6 through 9 refer to an election with 4 candidates (A, B, C, and D), 71 voters and preference schedule given by the following table.

| Table 1.1 | Number of voters | 8 | 19 | 2 | 15 | 27 |
|---|---|---|---|---|---|---|
| | First choice | A | D | D | B | C |
| | Second choice | B | A | B | D | A |
| | Third choice | D | B | A | A | D |
| | Fourth choice | C | C | C | C | B |

**(4 pts)** 6. Using the plurality method the winner of the election is

A) A.

B) B.

C) C.

D) D.

E) None of the above.

**(4 pts)** 7. Using the plurality with elimination method the winner of the election is

A) A.

B) B.

C) C.

D) D.

E) None of the above.

**(4 pts)** 8. Using the Borda count method the winner of the election is

A) A.

B) B.

C) C.

D) D.

E) None of the above.

**(4 pts)** 9. Using the method of pairwise comparisons the winner of the election is

A) A.

B) B.

C) C.

D) D.

E) None of the above.

**(2 pts)** 10. "If candidate or alternative X is the winner of an election and, in a reelection, all the voters who change their preferences do so in a way that is favorable only to X, then X should still be the winner of the election." This fairness criterion is called the

A) majority criterion.

B) Condorcet criterion.

C) monotonicity criterion.

D) independence of irrelevant alternatives criterion.

E) None of the above.

**(4 pts)** 11. If an honest coin is tossed twice, the probability that both tosses will come up heads is

_____

**(4 pts)** 12. If an honest coin is tossed twice, the probability that at least one of the tosses will come up heads is

_____

**(4 pts)** 13. An honest pair of dice is rolled. The probability of rolling a total of $7$ is

_____

Questions 14 through 18 refer to the following example: A computer password is made up of five characters. Each character can be a capital letter (A through Z) or a digit (0 through 9).

**(4 pts)** 14. How many different such computer passwords are there:

_____

**(4 pts)** 15. How many passwords do not start with the digit $0$ ?

_____

**(4 pts)** 16. How many passwords start with a digit?

_____

**(4 pts)** 17. How many passwords consist of five different letters?

_____

**(4 pts)** 18. Suppose that we roll a pair of honest dice $10$ times. If we roll a total of $7$ at least once we win. If we never roll a total of $7$ we lose. What is the probability that we win at this game?

_____

Questions 19 through 21 refer to the following situation: Three players (one divider and two choosers) are going to divide a cake fairly using the lone divider method. The divider cuts the cake into three slices ( $s_1$ , $s_2$ , and $s_3$ ).

**(2 pts)** 19. If the choosers declarations are Chooser 1: $\{s_2\}$ and Chooser 2: $\{s_3\}$, which of the following is a fair division of the cake?

A) Chooser 1 gets $s_1$ ; Chooser 2 gets $s_2$ ; Divider gets $s_3$ .

B) Chooser 1 gets $s_3$ ; Chooser 2 gets $s_2$ ; Divider gets $s_1$ .

C) Chooser 1 gets $s_2$ ; Chooser 2 gets $s_3$ ; Divider gets $s_1$ .

D) Chooser 1 gets $s_2$ ; Chooser 2 gets $s_1$ ; Divider gets $s_3$ .

E) None of the above.

**(2 pts)** 20. If the choosers declarations are Chooser 1: $\{s_2, s_3\}$ and Chooser 2: $\{s_1, s_3\}$, which of the following is not a fair division of the cake?

A) Chooser 1 gets $s_2$ ; Chooser 2 gets $s_3$ ; Divider gets $s_1$ .

B) Chooser 1 gets $s_1$ ; Chooser 2 gets $s_3$ ; Divider gets $s_2$ .

C) Chooser 1 gets $s_3$ ; Chooser 2 gets $s_1$ ; Divider gets $s_2$ .

D) Chooser 1 gets $s_2$ ; Chooser 2 gets $s_1$ ; Divider gets $s_3$ .

E) None of the above.

**(2 pts)** 21. If the choosers declarations are Chooser 1: $\{s_2\}$ and Chooser 2: $\{s_2\}$, which of the following is a fair division of the cake?

A) Chooser 1 gets $s_2$ ; Chooser 2 gets $s_1$ ; Divider gets $s_3$ .

B) Chooser 1 gets $s_2$ ; Chooser 2 gets $s_3$ ; Divider gets $s_1$ .

C) Chooser 1 gets $s_3$ ; Chooser 2 gets $s_1$ ; Divider gets $s_2$ .

D) Chooser 1 and Chooser 2 split $s_2$ ; Divider gets $s_1$ and $s_3$ .

E) None of the above.

Questions 22 through 25 refer to the following situation: Five players agree to divide a cake fairly using the last diminisher method. The players play in the following order: Anne first, Betty second, Cindy third, Doris fourth, and Ellen last. Suppose that there are no diminishers in round 1 and Cindy and Doris are the only diminishers in round 2.

**(2 pts)** 22. Which player gets her fair share at the end of round 1?

A) Anne
B) Betty
C) Cindy
D) Doris
E) Ellen

**(2 pts)** 23. Which player is the first to cut the cake at the beginning of round 2:

A) Anne
B) Betty
C) Cindy
D) Doris
E) Ellen

**(2 pts)** 24. Which player gets her fair share at the end of round 2?

A) Anne
B) Betty
C) Cindy
D) Doris
E) Ellen

**(2 pts)** 25. Which player is the first to cut the cake at the beginning of round 3?

A) Anne
B) Betty
C) Cindy
D) Doris
E) Ellen

Questions 26 through 29 refer to the following: Four players $(P_1, P_2, P_3, P_4)$ agree to divide the 16 items below using the method of markers.

Each of the player's three markers are placed as follows:

$P_1$ : immediately to the right of items 1, 6, 12

$P_2$ : immediately to the right of items 3, 8, 15

$P_3$ : immediately to the right of items 2, 9, 14

$P_4$ : immediately to the right of items 2, 7, 12.

**(2 pts)** 26. Item 5

A) goes to $P_1$.
B) goes to $P_2$.
C) goes to $P_3$.
D) goes to $P_4$.
E) is left over.

**(2 pts)** 27. Item 9

A) goes to $P_1$.
B) goes to $P_2$.
C) goes to $P_3$.
D) goes to $P_4$.
E) is left over.

**(2 pts)** 28. Item 11

A) goes to $P_1$.
B) goes to $P_2$.
C) goes to $P_3$.
D) goes to $P_4$.
E) is left over.

**(2 pts)** 29. Item 16

A) goes to $P_1$.
B) goes to $P_2$.
C) goes to $P_3$.
D) goes to $P_4$.
E) is left over.

Questions 30 through 33 refer to the following example: Three heirs ( $A$ , $B$ , and $C$ ) must divide fairly an estate consisting of two items – a house and a painting – using the method of sealed bids. The players' bids (in dollars) are:

| | $A$ | $B$ | $C$ |
|---|---|---|---|
| House | 195,000 | 210,000 | 199,000 |
| Painting | 45,000 | 15,000 | 53,000 |
| Totals | 240,000 | 225,000 | 252,000 |

**(2 pts)** 30. The original fair share of player $C$ is worth

A) \$252,000.
B) \$126,000.
C) \$84,000.
D) \$53,000.
E) None of the above.

**(2 pts)** 31. After the initial allocation to each player is made there is a surplus of

A) \$24,000.
B) \$32,000.
C) \$78,000.
D) \$0.
E) None of the above.

**(2 pts)** 32. After all is said and done, the final allocation to player $A$ is:

A) \$80,000 in cash.
B) \$88,000 in cash.
C) the painting and \$43,000 in cash.
D) the house and $A$ must pay \$107,000.
E) None of the above.

**(2 pts)** 33. After all is said and done, the final allocation to player $C$ is:

A) \$84,000 in cash.
B) \$92,000 in cash.
C) the painting and \$40,000 in cash.
D) the painting and $C$ must pay \$31,000.
E) None of the above.