Math 551 Exam III Spring 1999 Spring Fever

NAME _____

**1.** Let $A : \mathbb{R}^3 \to \mathbb{R}^3$; $\quad a = \begin{bmatrix} 4 & 0 & 4 \\ 1 & 2 & 0 \\ -3 & 0 & -4 \end{bmatrix}$ in the standard bases.

a) Compute the eigenvalues of $A$.

b) Compute a maximal set of independent eigenvectors. Is it a basis for $\mathbb{R}^3$?

**2.** Find the *set* of all $a$, $b$ and $c$ so that

$$\det \begin{pmatrix} 3 & \mathrm{a} & 2 \\ -1 & \mathrm{b} & 0 \\ 4 & \mathrm{c} & 7 \end{pmatrix} = 0.$$

**3.** Find five vectors, $v_1$, $v_2$, $v_3$, $v_4$ and $v_5$ in $\mathbb{R}^4$ so that $\{v_1, v_2, v_3, v_4\}$, $\{v_1, v_2, v_3, v_5\}$, and $\{v_1, v_2, v_4, v_5\}$, are the only subsets of $\{v_1, v_2, v_3, v_4, v_5\}$, which are bases for $\mathbb{R}^4$.

**4.** Let $A : \text{span}\{e_1, e_2\} \to \text{span}\{f_1, f_2\}$, $A = \begin{bmatrix} 3 & -2 \\ 7 & 5 \end{bmatrix}$, $e_1 = 2g_1 + g_2$, $e_2 = g_1 - g_2$, and $V = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$ in $\{g_1, g_2\}$. Compute $AV$.

**5.** If $x(t)$ is a vector, the solution to the differential equation $\ddot{x}(t) = Ax(t)$ is

$$x(t) = \cos(At)x_0 + \sin(At)x_1$$

where $x_0$ and $x_1$ are vectors which describe the initial conditions.

Let $V = \text{span}\{f_1, f_2\}$, $F = \{f_1, f_2\}$, $e_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ in the $F$ basis, and $E = \{e_1, e_2\}$.

The set $E$ is a basis of eigenvectors for the linear map, $A : V \to V$ given by $A = \begin{bmatrix} 4 & 3 \\ -2 & 11 \end{bmatrix}$ in $F \to F$.

a) Find the eigenvalues corresponding to $e_1$ and $e_2$.

b) Find the matrix for $A$ in the bases $E \to E$.

c) Compute $A^2$ and $A^3$ in the bases $E \to E$.

d) Compute $\sin(A) \equiv A - \frac{1}{3!}A^3 + \frac{1}{5!}A^5 - \ldots$ in the bases $E \to E$.

Hint: $\sin(5) = 5 - \frac{1}{3!}5^3 + \frac{1}{5!}5^3 - \ldots$.

e) Compute $\sin(A)$ in the bases $F \to F$.