Math 551 Name_____

Exam 2 July 23, 2001

1. Given that the eigenvalues of the matrix A are $\lambda = 2, -1$ find (if possible) a basis of eigenvectors. Is A diagonlizable? Express $X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as a linear combination of eigenvectors and use this to compute A^3X.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

2. Find the eigenvalues and a basis of eigenvectors for the following matrix B; find a diagonal matrix D and an invertible matrix Q so that $B = QDQ^{-1}$. Use this factorization to compute B^4.

$$B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

3. Compute (show your work!) $\det A$ using row reduction. If A is invertible, find $\det A^{-1}$.

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 2 & 1 & 5 \\ 2 & 0 & 0 & 1 \\ 7 & 4 & 0 & -3 \end{bmatrix}$$

4. Show that the following vectors form an orthogonal basis for $\mathbb{R}^3$.

$$Q_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad Q_2 = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix} \qquad Q_3 = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}.$$

Find the coordinate vectors of $X = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ with respect to this basis.

5. Find all possible 2×2 matrices with eigenvalues $1, 2$ and corresponding eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

6. Please answer True or False to each of the following questions. Please show your reasoning! A correct answer without explanation will receive only half credit.

   (a) The following matrix is orthogonal.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

   (b) Suppose that $\det A = 2$, $\det(A + I) = 3$, and $\det(A + 2I) = 5$. Then,

$$\det(A^4 + 3A^3 + 2A^2) = 45.$$

   (c) If the following matrix is invertible, then the $(1, 4)$ entry of A^{-1} is zero.

$$A = \begin{bmatrix} 1 & 2 & 1 & 13 \\ 2 & 3 & -5 & -1 \\ 0 & 5 & -4 & 12 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

   (d) Suppose that A is diagonalizable over $\mathbb{R}$. Then the eigenvalues of A^2 are non-negative.