# MATH 551 EXAM 2

10:30-11:20 am, Friday March 17, 2006

**Name:**_____

No books or formula sheets are allowed. Use the back page as a sketch paper. For full credit, show your work in detail.

| Total: 100 | #1 | #2 | #3 | #4 | #5 |
|---|---|---|---|---|---|
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**1 (20 pts).** We have shown that the functions $\{1, x, x^2, x^3\}$ are linearly independent. Let $\mathcal{S}$ be the linear space spanned by these functions.

**a.** For which number $n$ is this space $\mathcal{S}$ isomorphic to the space $\mathbf{R}^n$?

**b.** Ler $D$ be the differential operator defined on the space $S$, (i.e., if $y = y(x) \in S$, then $D(y) = y'$.) Find the matrix $M$ representing $D$ with respect to the basis $\{1, x, x^2, x^3\}$.

**2 (20 pts).** Let $A = \begin{bmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ b & c & 0 \end{bmatrix}$ where $a, b, c$ are real numbers.

**(i).** Find $A^2$ and $A^3$.

**(ii).** Show that the inverse $(I - A)^{-1} = I + A + A^2$.

**3 (20 pts).** An economy system consists of three sectors E, F and G. The consumption matrix C is given by

| | E | F | G |
|---|---|---|---|
| E | 0.2 | 0.1 | 0.5 |
| F | 0.6 | 0.2 | 0.2 |
| G | 0.1 | 0.1 | 0.1 |

If the external demand is $D = [50, 60, 70]^t$, find the production vector $X = [l, m, n]^t$.

**4(20 pts).** For the following ordered basis $\mathcal{B} = \{X_1 = [1, 3, 2]^t, X_2 = [-1, 1, -1]^t, , X_3 = [5, 1, -4]^t\}$ in $\mathbf{R}^3$.

**(a).** Find the point transform matrix $P_{\mathcal{B}}$ and coordinate transform matrix $C_{\mathcal{B}}$

**(b).** Let $X = [21, 21, 21] \in \mathbf{R}^3$, find its $\mathcal{B}$-coordinate.

**5(20 pts).** Suppose that $\begin{vmatrix} p & q & r \\ s & t & u \\ x & y & z \end{vmatrix} = 3.$

Apply the reduction propertie of determinants to compute the following determinant:

$$\begin{vmatrix} 2p & 2q & 2r \\ 3s-p & 3t-q & 3u-r \\ 4x+3p & 4y+3q & 4z+3r \end{vmatrix}$$

Must show your work in details.