Lab time ____________ Name ______________________________

Applied Matrix Theory - MATH 551

Exam 2

April 7, 2003

Show all your work in the space provided. Each problem is worth 10 points.

1(a) Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which rotates the plane by 120° counterclockwise.

(b) Let A be the matrix associated to the transformation T in (a). Explain why A is an orthogonal matrix.

2. Draw the image of the unit square $S = \{(s,t) : 0 \leq s \leq 1,\, 0 \leq t \leq 1\}$ under the transformation $T(x) = Ax$ where $A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$. Label the vertices on the figure you draw.

3. Consider the transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by $T(x) = Ax$ where $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 5 & 1 \\ 2 & 0 & 2 & -2 \end{bmatrix}$. Find a basis for the nullspace of T and a basis for the image of T.

4. Find $proj_W v$ where $W = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$ in $\mathbb{R}^4$ and $v = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$.

5. Given $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ find A^{-1}.

6. Find an LU decomposition of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 4 & 3 & 3 \\ 6 & 0 & 2 \end{bmatrix}$.

7(a) Find the coordinates of the polynomial $2x^2 + x + 1$ with respect to the basis $\{x^2, x+1, 2\}$ of P_2.

(b) Let $\mathcal{F}$ be the vector space of all continuous real valued functions in $\mathbb{R}$. Let $W = \text{span}\{e^x, e^{-x}\}$; this is a subspace of $\mathcal{F}$. (You may assume this.) Find the coordinates of the function $\sinh x$ with respect to the given basis of W.

8. Each row of the table below summarizes data collected for some matrix A and the corresponding transformation. The "always solvable" column refers to whether or not the system $Ax = b$ is solvable for all b and the "unique solution" column refers to whether or not there will be at most one solution. Fill in the missing data from the table.

size	always solvable	Unique solution	dimension of image	dimension of nullspace	rank
3x4	yes				
4x5			3		

9. True or false. Give a brief justification.

A. ______ If A is an $n \times n$ matrix and if $A \neq 0$ then $A^2 \neq 0$.

B. ______ Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation. Then the image of T is a subspace of $\mathbb{R}^m$.

C. ______ Suppose A is an $m \times n$ matrix. Then rank $A = \dim(\text{rowspace } A)$.

D. ______ If $T : \mathbb{R}^n \to \mathbb{R}^n$ is given by $T(x) = Ax$ and T is one to one and onto then A is invertible.

E. ______ Suppose A is a matrix. Then $\dim(\text{columnspace } A) + \dim(\text{nullspace } A)$ $=$ the number of columns of A.

10. Suppose $S : V \to W$ is a linear transformation from the vector space V to the vector space W. Prove:

(a) $S(0) = 0$

(b) The nullspace of S is a subspace of V.