Name _____

Applied Matrix Theory
Math 551
Exam 2
April 2, 2001

Show all your work in the space provided under each question. Each problem is worth 10 points.

1. The vectors below form a spanning set for a subspace of $M(3,1)$. Find a basis for that subspace. What is its dimension?

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ 3 \\ 8 \end{bmatrix}$$

2. Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects points in the y axis. Find the matrix which describes this transformation.

3. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Find A^{-1}.

4. Define $T : \mathbb{R}^3 \to \mathbb{R}^5$ by $T(x) = Ax$ where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Find a basis for the image of T. Find dim (image T).

5. Suppose $T : V \to W$ is a linear transformation of the vector space V to the vector space W. Suppose $\{V_1, V_2, V_3\}$ is a dependent set of vectors in V. Show that the set $\{T(V_1), T(V_2), T(V_3)\}$ is dependent in W.

6. Find a transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ which rotates $\mathbb{R}^3$ by $\frac{\pi}{4}$ radians around the z-axis. (When viewed from the positive z-axis.)

7. Define $T : \mathbb{R}^4 \to \mathbb{R}^3$ by $T(v) = Av$ where

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{bmatrix}.$$

Find a basis for the image of T and a basis for the nullspace of T.

8. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with the property that $T(x) = b$ always has a solution for every $b \in \mathbb{R}^m$. What can you say about the relative sizes of m and n? Explain your answer.

9. Find the LU decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & 2 & 6 \end{bmatrix}.$$

10. Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ whose image is the line $y = 4x$.