From maginnis@math.ksu.edu Fri Dec 19 15:45:01 2003 Date: Fri, 19 Dec 2003 From: John Maginnis ¡maginnis@math.ksu.edu¿ To: sheree ¡math@math.ksu.edu¿ Sub 2

Dear Sheree, I think I forgot to give you this second exam.
John Maginnis

Name _____ Math 551, Applied Matrix Theory

Signature _____ Exam 2, November 7, 2003

Show all work for full credit. The exam is worth 100 points.

(6 pts) 1. Find a 2×2 matrix A such that the linear transformation of the plane defined by $T(\vec{v}) = A\vec{v}$ is a counterclockwise rotation by 90 degrees.

(6 pts) 2. Show that the functions $f(x) = x^5$ and $g(x) = x^6$, with domain $[-1, 1]$, are perpendicular.

(9 pts) 3. Use row reduction to find the inverse matrix for the matrix

$$A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}.$$

(7 pts) 4. Let A be a square matrix. Assume that rank(A^2)=rank(A). Show that the nullspace of A^2 is equal to the nullspace of A.

(14 pts) 5. Let A be the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 0 & 2 & 1 \end{pmatrix}$.

(a) Use row reduction to find the $A = LU$ factorization, with L a lower triangular matrix and U an upper triangular matrix.

(b) Use the $A = LU$ factorization to solve the system of equations

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}.$$

(9 pts) 6. Find the coordinate vector of the polynomial $f(x) = 2x^2 + 3x + 2$ with respect to the ordered basis $B = \{1, x-2, (x-2)^2\}$ for the vector space of polynomials of degree less than or equal to two.

7. Let $\vec{v}_1 = (0, 1, 1), \vec{v}_2 = (1, 1, 3)$, and $\vec{v}_3 = (3, 1, 1)$.

(5 pts) (a) Find the projection of the vector $\vec{v}_2$ onto the line spanned by the vector $\vec{v}_1$.

(7 pts) (b) Find the projection of the vector $\vec{v}_3$ onto the plane spanned by the two vectors $\vec{v}_1$ and $\vec{v}_2$.

(5 pts) (c) Give the orthogonal basis for R^3 determined by the Gram-Schmidt algorithm.

(12 pts) 8.

(a) Let A be a 5×3 matrix, with entries in the real numbers, and let $T : R^3 \to R^5$ be the linear transformation $T(\vec{v}) = A\vec{v}$. Assume that T is one-to-one. Find the rank of the matrix A.

(b) Let B be a 3×5 matrix, with entries in the real numbers, and let $T : R^5 \to R^3$ be the linear transformation $T(\vec{v}) = B\vec{v}$. Assume that T is onto. Find the nullity of the matrix B.

(8 pts) 9. Let V be the vector space of continuous functions $f : R \to R$, with domain and range the real numbers. Determine whether the function $T : V \to R$ defined by the integral below is a linear transformation.

$$T(f) = \int_0^1 x^2 f(x) dx$$

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(12 pts) 10. A small country has only two industries : agriculture and factory manufacturing. Assume that .3 units of the agricultural output is consumed per unit of food produced by the farmers, and .2 units of the agricultural output is consumed per unit of manufacturing produced by the factory workers. Assume that .1 units of the manufacturing output is consumed per unit of food produced by the farmers, and .4 units of the manufacturing output is consumed per unit produced by the factory workers. The country wants to export 500 units of food and 100 units of manufacturing every day.

(a) Find the consumption matrix.

(b) Give a system of linear equations describing the output of this economy.

(c) Find the level of output required to satisfy the export demand.