Name _____

Applied Matrix Theory - MATH 551
Exam 2
November 4, 2002

Show all your work in the space provided under each question. each problem is worth 10 points.

1. Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which rotates points by $\frac{\pi}{3}$ radians counter-clockwise, expressing your answer in simplest form.

2. Find the coordinates of the polynomial $p(x) = x^2 + 2x + 2$ with respect to the basis $\{1, (x+1), (x+1)^2\}$ of $\mathbb{R}$.

3. Consider the matrices $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 5 & 1 \\ 1 & 6 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ in $M(2,2)$. Find a basis for the subspace they span.

4. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^{-1}.

5. Find the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

6. Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $Tx = Ax$. Find the image of the unit circle $x^2 + y^2 = 1$ under this transformation.

7. Let $A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 1 & 1 & 2 & 3 \\ 2 & 0 & 2 & 2 \end{bmatrix}$ and define $T : \mathbb{R}^4 \to \mathbb{R}^3$ by $Tv = Av$.

Find a basis for the image of T and a basis for the nullspace of T.

8. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with the property that $Tx = b$ has exactly one solution for each $b \in \mathbb{R}^m$. What is the rank of T? What can you say about the relative sizes of m and n? Explain.

9. Consider the space $C^\infty(\mathbb{R})$ of infinitely differentiable functions on $\mathbb{R}$ and the mapping $D : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$ given by $D(f) = \dfrac{df}{dx}$. Consider the following statements. Circle the letter of each that is true. (No justification is required.)

A. D is a linear transformation.

B. The space $P_6 =$ all polynomials of degree ≤ 6 is a subspace of $C^\infty(\mathbb{R})$.

C. D is one-to-one.

D. $D(0) = 0$

E. D maps P_6 into itself.

10. True or false. (No justification required.)

1. If V is a vector space and if there are vectors $v_1, \ldots, v_k$ which span V, then V has dimension k. _____

2. The rank of a matrix plus the dimension of the nullspace = the number of columns of the matrix. _____

3. Any matrix A can be written in the form $A = LU$ where L is lower triangular and U is upper triangular. _____

4. If $v_1, \ldots, v_n$ are linearly independent vectors in a vector space V then $\dim V = n$. _____

5. If V and W are vector spaces and $T : V \to W$ is linear, then $T(0) = 0$. _____