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Applied Matrix Theory - MATH 551
Exam 2
November 6, 2000

Show all your work in the space provided under each question. Each problem is worth 10 points.

1. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ Find A^{-1}.

2. Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects points in the y-axis. Find the matrix which describes this transformation.

3. Define $T : \mathbb{R}^4 \to \mathbb{R}^3$ by $T(v) = Av$ where $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 0 & 2 & 2 \\ 1 & 2 & 3 & 5 \end{bmatrix}$.

Find a basis for the image of T and a basis for the nullspace of T.

4. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with the property that $T(x) = b$ always has a solution for every $b \in \mathbb{R}^m$. If $T(x) = Ax$ for all $x \in \mathbb{R}^n$, that is, T is defined by A, what is the rank of A? What can you say about the relative sizes of m and n? Explain your answers.

5. Suppose A has an LU factorization where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

Use this to solve $Ax = y$ where $y = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

6. Suppose that $T : V \to W$ is a linear transformation from the vector space V to the vector space W. Show that the image of T is a subspace of W.

7. Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which sends the unit circle $x^2 + y^2 = 1$ to the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$.

8. Find the coordinates of the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ with respect to the basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ of $\mathbb{R}^3$.

9. Find the projection of the vector $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ onto the subspace of $\mathbb{R}^4$ spanned by the orthogonal vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$.

10. Use the Gram-Schmidt process to find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$.