MATH 551 EXAM 1

10:30–11:25 pm, Friday February 24, 2006

Name:_____

No books or formula sheets are allowed. Use the back page as a sketch paper. For full credit, show your work in detail.

Total: 100	#1	#2	#3	#4	#5

1 (20 pts). Let $A = \begin{bmatrix} 1 & -2 & 1 & 2 \\ -1 & 3 & 2 & -2 \\ 0 & 1 & 3 & 0 \\ 1 & 2 & 13 & 5 \end{bmatrix}$.

(a. 5pts). Find a basis of the column space $\mathcal{S}_C$ of matrix A.

(b. 5pts). Find a basis of row space $\mathcal{S}_R$ of matrix A.

(c. 5pts). What is the dimension of null space $\mathcal{N}(A)$?

(d. 5pts). Is A a nonsigular matrix?

2 (20 pts). Let the set $W = \{[a+b+2c, 2a+b+3c, a+b+c]^t\}$, for all possible $a, b, c \in \mathbf{R}$.

(i). Show that W is a subspace of $\mathbf{R}^3$. (ii) Find a basis of the W.

3 (20 pts). Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 3 & 0 \\ 1 & 0 & 2 & 1 \\ 3 & 3 & 8 & 2 \end{bmatrix}$.

(a.) Does vector $B = [1, 2, 3, 4]^t$ belong to the column space $\mathcal{S}_C$ of matrix A ? Give your reason.

(b.) Find a basis of the null space $\mathcal{N}(A)$

4(20 pts). Show that the set of elements $\{1, e^x, e^{2x}, e^{3x}\}$ are linearly independent.

5(20 pts). In the downtown section of a city, two set of one-way streets intersect as shown in the following figure. The average volume of traffic entering and leaving this section during rush hour is given in the digram.

(1.) Write out the system of equations and solve for x_1, x_2, x_3, x_4.

(2). Order these x_1, x_2, x_3, x_4 from the highest traffic flow to the lowest.