Lab time ____________ Name ______________________________

Applied Matrix Theory - MATH 551

Exam 1

Friday, February 21, 2003

Show all your work in the space provided under each question. Each problem is worth 10 points.

1. Indicate each set on a graph, and clearly label any vertices or intercepts of the figures you draw.

a) $\{(2,1) + t(-3,3) : t \in \mathbb{R}\}$

b) $\{t(1,-1) : t > 0\}$

2. In $\mathbb{R}^2$ let $A = (-1, 1)$. Give a geometric description of the set of all vectors B in $\mathbb{R}^2$ such that $B \cdot A \leq \frac{\sqrt{2}}{2}\,|B|$.

3. Solve the system
$$\begin{aligned} 3x + 2y + z &= 4 \\ x + y &= 3 \\ x + y + z &= -1 \end{aligned}$$

4. In $M(1,3)$ are the matrices $[1\ 2\ 3]$, $[5\ 0\ 2]$ and $[4\ \ -2\ 2]$ independent? Explain.

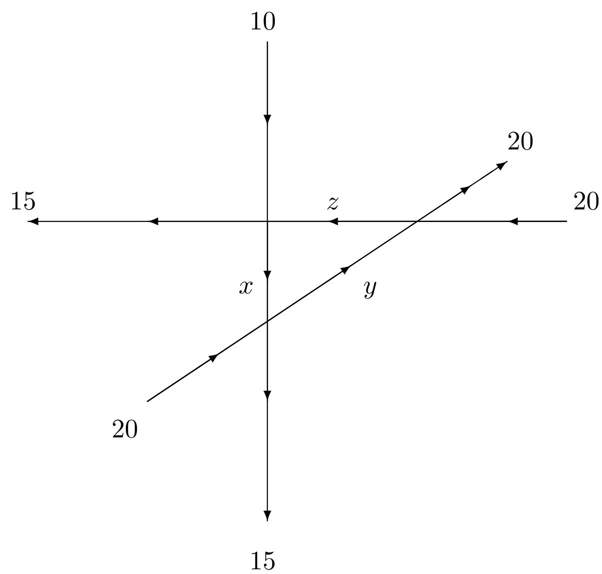
5. Find the nullspace of the matrix $A = \begin{bmatrix} 3 & 1 & 4 & 2 \\ 1 & 1 & 3 & 2 \\ 1 & 0 & 3 & 0 \end{bmatrix}$

6. Suppose $\{x, y\}$ are a basis for a subspace W of a vector space V. Prove that $u = x + y$ and $v = x - y$ are also a basis for W.

7. Find a basis for the columnspace of the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 6 & 3 & 3 \\ 1 & 2 & 1 & 2 \\ 2 & 12 & 6 & 8 \end{bmatrix}$

8. In $\mathbb{R}^2$ let $u = (2, 1)$ and $v = (1, 2)$. Set $S = \{su + tv : 0 \leq s \leq 1, -1 \leq t \leq 1\}$. Draw S. Label all vertices and intercepts on the figure you draw.

9. Consider the network of streets in the center of the city of Townsville depicted below. Each street is labeled with the direction traffic must travel as well as the number of cars entering and exiting per hour along this road. Find a system of equations which describes this network. If $z = s$, express y and x in terms of s. If $s = 20$, how many cars are at y?



10. Answer True or False. Give a brief justification. In some cases if the statement is true, you may simply say "this is a theorem from class" (if that is the case), and in other cases you might give a very short explanation. If the statement is false, show by example that it is false.

_____ A) A homogeneous system is inconsistent if it has more equations than variables.

_____ B) A system of equations with more variables than equations is always solvable.

_____ C) In an n dimensional space it is impossible to find $n+1$ linearly independent vectors.

_____ D) The nullspace of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.

_____ E) The rank of a matrix A is the same as the dimension of the rowspace of A.