From maginnis@math.ksu.edu Mon Sep 29 10:28:48 2003 Date: Mon, 29 Sep 2003
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551 last week

Name ______________________ Math 551, Applied Matrix Theory

Signature ____________________ Exam 1, September 26, 2003

Show all work for full credit. The exam is worth 100 points.

(12 pts) 1. Consider the system of equations $\{x - 2y = 4,\ 3x + y = 5\}$.

(a) Rewrite the system using matrix notation.

(b) Solve the system.

(15 pts) 2. Let $A = \begin{pmatrix} 1 & 2 & 0 & 1 & 2 \\ 2 & 4 & 1 & 4 & 5 \\ 1 & 2 & 1 & 3 & 3 \end{pmatrix}$.

(a) Find the reduced row echelon form of the matrix A.

(b) Give a basis for the column space of A.

(c) Give a basis for the nullspace of A.

(9 pts) 3. Find equations describing the plane containing the three points

$$A = (1, 1, 2),\ B = (2, 3, 1),\quad \text{and } C = (1, 2, 3).$$

(6 pts) 4. In the plane $R^2$, shade the region

$$X = \{\ s(-1, 1) + t(2, 0)\ \Big|\ 0 \le s \le 1 \ \mathit{and}\ 0 \le t \le 1\ \}.$$

(15 pts) 5. Show the three matrices $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$, $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, and $\begin{pmatrix} -1 & 1 \\ 4 & -1 \end{pmatrix}$ are linearly dependent.

(6 pts) 6. Give a normal vector to the plane $2(x-1)+3(y-2)+5(z-3)=0$.

(9 pts) 7. Show the set of smooth functions $y=f(x)$ which satisfy the differential equation $y''-3y'+2y=0$ is a vector space (closed under linear combinations $c_1f(x)+c_2g(x)$.)

(12 pts) 8. The 3 one-way roads Apple, Orange, and Lemon Streets meet at 3 intersections, forming a triangular hub with a counterclockwise traffic flow. Measurements are taken on most of the roads leading toward or away from the hub. 10 cars per minute are driving toward the hub on Apple Street, 15 cars per minute approach the hub on Orange Street, and 8 cars per minute approach the hub on Lemon Street. 5 cars per minute drive away from the hub on Apple Street and 10 cars per minute leave the hub on Lemon Street.

(a) Give 3 equations in 4 variables describing the unknown traffic flows.

(b) Show that 18 cars per minute drive away from the hub on Orange Street.

(8 pts) 9. Assume that the three vectors $\{v_1, v_2, v_3\}$ are linearly dependent, but that any two of these vectors are linearly independent. Show that $span\{v_1, v_2\} = span\{v_1, v_3\}$.

(8 pts) 10. Let $A\vec{x} = \vec{b}$ be a nonhomogeneous system of m linear equations in n variables (so A is an $m \times n$ matrix). Let $M = (A\ \vec{b})$ be the augmented matrix, with $n + 1$ columns. Assume that $rank(M) = rank(A)$. Show that there must exist at least one solution to the system $A\vec{x} = \vec{b}$.