Name _____

Applied Matrix Theory - MATH 551
Exam 1
September 27, 2002

Show all your work in the space provided under each question. each problem is worth 10 points.

1. Find all solutions to the system of equations. (Show your steps.)

$$\begin{aligned} x + y - 3z &= -1 \\ 3x - y + z &= 7 \\ 5x + 3y - 5z &= z \end{aligned}$$

2. Let $x = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$. Is there any element in their span with all positive entries? Prove your answer.

3. Let $A = (2, 0)$. Give a geometric description of all vectors B in $\mathbb{R}^2$ such that $B \cdot A \geq |B|$.

4. Are the matrices $\begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 3 & 0 \\ 2 & -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ linearly independent in $M(2,2)$?

5. Find a spanning set for the nullspace of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 5 \\ 2 & 1 & 3 \end{bmatrix}$.

6. Let $S = \{A, B, C, D\}$ be a set of $m \times n$ matrices. Suppose that $D = 2A + B + C$ and $C = B - A$.

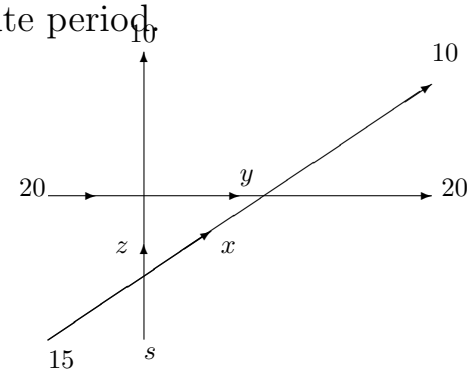
(a) Is the set $\{A, B, D\}$ independent? Explain.

(b) Is the set $\{A, C, D\}$ independent? Explain.

7. Is the matrix $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ in the span of the matrices $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ -1 \\ -2 \end{bmatrix}$?

Prove your answer.

8. Consider the network of roads pictured below. Each road is labeled with the direction of traffic flow as well as the number of cars using that road in a ten-minute period.



Let x, y, z denote the number of cars passing the points labeled x, y, z on the diagram. Write a system of equations which describes this network. Solve this expressing your answer as $z = s$, with expressions for x and y in terms of s. If $z = s$ what is x and y?

9. Draw a picture of the set $\{sv + tu : -1 \leq s \leq 1,\ -1 \leq t \leq 1\}$ where $v = (1, 0)$ and $u = (1, 1)$ in $\mathbb{R}^2$. Label vertices and intercepts of the figure you find.

10. True or false. No justification is required.

_____ A) In $\mathbb{R}^2$ the span of $(1, 2)$ is a line.

_____ B) If $\mathcal{S}$ and $\mathcal{T}$ are subspaces of a vector space V then so is $\mathcal{S} \cap \mathcal{T}$.

_____ C) If $\mathcal{S}$ and $\mathcal{T}$ are subspaces of a vector space V then so is $\mathcal{S} \cup \mathcal{T}$.

_____ D) If $\mathcal{S}$ and $\mathcal{T}$ are subspaces of a vector space V then so is $\mathcal{S} + \mathcal{T}$.

_____ E) In $\mathbb{R}^3$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ where $a \neq 0$, $b \neq 0$, $c \neq 0$ are linearly dependent.

_____ F) The nullspace of an $m \times n$ matrix is a subspace of $\mathbb{R}^n$.

_____ G) The column space is the same as the nullspace of a matrix.

_____ H) A homogeneous system $Ax = 0$ is always consistent.

_____ I) $Ax = b$ is solvable if and only if b is in the column space of A.

_____ J) The nullspace of an $m \times n$ matrix is a subspace of $\mathbb{R}^m$.