Name _____

Applied Matrix Theory - MATH 551
Exam 1
September 29, 2000

Show all your work in the space provided under each question. Each problem is worth 10 points.

1. Solve (using Gaussian elimination - show your steps) the system of equations.

$$\begin{aligned} x + 2y - 3z &= -1 \\ 3x - x + z &= 7 \\ 5x + 3y - 5z &= 2 \end{aligned}$$

2. Solve the system of equations.

$$\begin{aligned} x + y - z &= 0 \\ x + 2y + z &= 0 \\ 3x + 4y - z &= 0 \end{aligned}$$

3. Let $S = \{p(x) : p(x) \text{ is a polynomial of degree} \leq 2 \text{ and } p(1) = 0\}$.

Show that S is a subspace of $P_2 =$ the set of polynomials of degree ≤ 2.

4. Find a basis for the nullspace of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \end{bmatrix}$.

5. Given that $A = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ find all B in $\mathbb{R}^2$ so that $B \cdot A \geq |B|$.

6. Determine if the vectors $(1, 2, 1)$, $(3, 1, 2)$ and $(6, 7, 0)$ are linearly independent.

7. Given that $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 7 & 1 \end{bmatrix}$, $x = \begin{bmatrix} 9 \\ 0 \\ 1 \end{bmatrix}$, find Ax.

8. The matrices below form a spanning set for some subspace of $M(3,1)$. Find a basis for this subspace and its dimension.

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

9. Determine if x is in the row space of A.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \qquad x = \begin{bmatrix} 4 & 3 & 1 \end{bmatrix}$$

10. Let $x = [-1\ 1\ 0]$, $\quad y = [-1\ 0\ 1]$. Are there any elements in their span with all positive entries?