Math 551 Final Exam Spring 1999

NAME _____

1. (a) Compute $\begin{bmatrix} 5 & 1 \\ -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 7 & 4 \end{bmatrix}$

(b) If $x = ar\cos\theta + \delta z$ and $y = br\sin\theta$, then

$$\begin{aligned} dx &= a\cos\theta\, dr - ar\sin\theta\, d\theta + \delta\, dz \\ dy &= b\sin\theta\, dr + br\cos\theta\, d\theta \qquad dz = dz. \end{aligned}$$

Find $dx \wedge dy \wedge dz$.

Row reducing $\begin{bmatrix} 5 & 3 & -1 & 2 & 7 \\ 2 & 3 & 2 & 4 & 0 \\ -1 & -2 & 3 & 2 & 1 \\ 6 & 4 & 4 & 8 & 8 \end{bmatrix}$ gives $\begin{bmatrix} 1 & 0 & 0 & \frac{56}{43} & -\frac{14}{43} \\ 0 & 1 & 0 & -\frac{8}{43} & \frac{2}{43} \\ 0 & 0 & 1 & \frac{42}{43} & \frac{11}{43} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(c) What is the set of all solutions of $\begin{aligned} 5x_1 + 3x_2 - x_3 + 2x_4 &= 7 \\ 2x_1 + 3x_2 + 2x_3 + 4x_4 &= 0 \\ -x_1 - 2x_2 + 3x_3 + 2x_4 &= 1 \\ 6x_1 + 4x_2 + 4x_3 + 8x_4 &= 8 \end{aligned}$?

(d) What is $\det \begin{bmatrix} 5 & 3 & -1 & 2 \\ 2 & 3 & 2 & 4 \\ -1 & -2 & 3 & 2 \\ 6 & 4 & 4 & 8 \end{bmatrix}$?

2. Pick the letters of the graphs which match each of the following functions:

i) $f(x, y) = 5x^2 + 8xy + y^2$ _____

ii) $g(x, y) = -3x^2 + 2xy - 4y^2$ _____

A. B. C.

3. Let $L : P_2 \to P_2$ be a differential operator of the form:

$$L(f(x)) = (a_1x^2 + a_2x + a_3)f''(x) + (a_4x + a_5)f'(x) + a_6f(x)$$

If the matrix for L is $\begin{bmatrix} 2 & 3 & -2 \\ 0 & 3 & 6 \\ 0 & 0 & 6 \end{bmatrix}$ in the bases $\{1, x, x^2\} \to \{1, x, x^2\}$,

find the constants a_1, a_2, a_3, a_4, a_5 and a_6.

4. Let $T : P_3 \to P_3$ be a linear map so that

$$\begin{aligned} T(x^3 - 2x) &= 4x^2 + 1, \\ T(4x^2 + 1) &= x^3 - 2x, \\ \text{and} \quad T(x^3 - 1) &= x^2 + x. \end{aligned}$$

Find $T(2x^3 - 4x^2 - 2x - 2)$.

5. (a) In some applications of linear algebra, one is given the eigenvalues and must find the matrix. Find constants a and b so that the eigenvalues of $\begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}$ are -2 and -3.

(b) What is the eigenvalue of $\begin{bmatrix} 5 & -4 & 2 & 2 \\ 3 & 7 & -16 & 2 \\ -1 & 1 & 6 & -2 \\ -1 & 1 & 6 & -2 \\ 4 & 2 & -13 & 5 \end{bmatrix}$ corresponding to $\begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix}$?

6. Find a basis, B, for $V = \{f(x) \in P_4 | f(2) = f(0),\ f(1) = 0\}$, so that the coordinates of $x^3 - 3x^2 + 2x$ are $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ in B.

7. If $a_{n+1} = 5a_n + 4b_n$ and $b_{n+1} = 3a_n + b_n$, $a_0 = 1$, $b_0 = 1$, what is a_{20}?

We will answer this in several steps. Note that $\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix}$.

Let $A = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$.

(a) Find the eigenvalues of A.

(b) Find a basis of eigenvectors for A, say, E.

(c) Compute the matrix for A in the bases $E \to E$.

(d) Compute A^{20} in the bases $E \to E$.

(e) Change bases to compute the matrix for A^{20} in the standard bases.

(f) Compute a_{20}.

8. Make up a linear algebra question, then solve it. Clearly, you should get the correct answer.

Extra points will be given for questions which use more than one concept or have applications.

State the question and the answer separately, e.g., "$2(3,-1)+4(2,2)=(14,6)$" is a fact, not a question and answer.

"Find $2(3,-1)+4(2,2)$." is a math question.

The answer is "$2(3,-1)+4(2,2)=(6,-2)+(8,8)=\boxed{(14,6)}$."

Have a great break!