MATH 551 FINAL EXAM

11:50am –1:40pm, Monday May 8, 2006

Name:____________________________

No books are allowed. Use the back of the page as sketch paper. For full credit, show your work in detail.

Total 200	Pg 1	Pg2	Pg3	Pg4	Pg5	Pg6	Pg7	Pg8

1 (10 pts). Let $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{bmatrix}$.

- (4pts). Find a basis for row space $\mathcal{S}_R$.
- (4pts). Fina a basis for the column space $\mathcal{S}_C$.
- (2pts). Find the rank of matrix A.

2 (15 pts, 3pts each.). If the eigenvalues of a matrix $A \in M(3,3)$ are $0, 3, -5$ Answer the following questions.

- **a** Is A singular?
- **b**. Is A diagonalizable?
- **c** If is, give the diagonalization matrix D=.
- **d.** Find $\det(A^t A) =$
- **e**. Find $\det(A + 2I) =$.

3 (12 pts). Find the spanning vectors of the orthogonal complement $\mathcal{S}^{\perp}$ of the subspace $\mathcal{S}$ =span$\{[1,1,1,1,1]^t, [1,1,0,1,-1]^t, [1,-1,0,0,1]^t\}$.

4. (13 pts). It is known that the functions$\{1, x, x^2, x^3\}$ are lineraly independent. Let $\mathcal{W}$ be the linear space spanned by these functions.

(**3pts**). For which n is $\mathcal{W}$ isomorphic to the space $\mathbf{R}^n$?

(**10 pts**). Let D be the differential operator definded on the space $\mathcal{W}$**:** if $y = y(x) \in \mathcal{W}$, then $D(y) := y'(x)$. Find the matrix M representing the operator D with respect to the above basis $\{1, x, x^2, x^3.\}$ (Hint: Write into $y = a \cdot 1 + b \cdot x + c \cdot x^2 + d \cdot x^3$.)

5 (13 pts). Find an orthogonal basis using Gram-Schmidt process for the subspace
$\mathcal{B}$ =span$\{[1,2,1,1]^t, [-2,1,1,-1]^t, [1,2,1,3]^t\}$

6 (12 pts). Let $f(x) = x$. Find its best approximation $f_0(x)$ by the orthoganal functions $\sin \pi x, \sin 2\pi x, \sin 3\pi x$ in the function space $\mathcal{F}[-1,1]$.

7 (10 pts). Suppose that $\begin{vmatrix} p & q & r \\ s & t & u \\ x & y & z \end{vmatrix} = 3.$

Apply related properties of determinant to compute the following determinant :

$$\begin{vmatrix} 2p & 2q & 2r \\ 3s-p & 3t-q & 3u-r \\ 4x+3p & 4y+3q & 4z+3r \end{vmatrix}$$

8 (15 pts, 3pts each). Let matrices $A, B \in M(n, n)$ and k be an integer. In ecah of the follwoing questions select true or false. (3pts each)

- **1.** $\det(AB) = \det(BA)$ true___false___
- **2.** $\det(cA) = c\det(A)$ true___false___
- **3.** $\det(A) = \det(B)$ implies $A = B$. true___false___
- **4.** $\det(A^k) = [\det(A)]^k$ true___false___
- **5.** If $A\vec{X} = \vec{\mathbf{0}}$ for some $\vec{X} \neq 0$ in R^n, then $\det(A) = 0$. true___false___

9. (10 pts). Apply Cramer's rule solve the following system for x (only) in terms of a, b, c. You need to simplify your formula.

$$\begin{array}{rrrcl} -x & +2y & -3z & = & a \\ 3x & +y & -z & = & b \\ 2x & +3y & +5z & = & c \end{array}$$

10 (15 pts). It is known that the matrix $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ has three eigenvectors

$X = [-1, 2, 0]^t,\ Y = [0, 0, 1]^t,\ Z = [1, 0, 2]^t.$

- **(5pts).** Find the corresponding eigenvalues.
- **(10pts).** Let vector $B = [0, 2, 3]^t$. Find $A^{100}B$.(You do not have to carry out the value of a power, say, 3^{100}. The power denotes the value itself.)

11. (13 pts). Given that $\lambda_1 = 0, \lambda_2 = \lambda_3 = 1$ are eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}.$$

(10 pts.) Find the corresponding eigenvectors.

(3pts.). Is A diagonalizable ? State your argument.

12 (12 pts). Let $A = \begin{bmatrix} 0 & x & y \\ 0 & 0 & z \\ 0 & 0 & 0 \end{bmatrix}$. **(5pts).** Find A^3.

(7pts.) Show that the inverse $(I - A)^{-1} = I + A + A^2$

13 (15 pts). Suppose that the total pupulation of a large metropolitan area remains relatively fixed. Each year 6% of the people living in city move to suburbs and 2% of the residents in suburb move to the city.

(a. 5pts.). Describe the transition matrix P for this population immigration.

	city	suburbs
city		
suburbs		

$P =$

(b. 10 pts.) Over the longt term, what percentage of population will live in city and suburbs, respectively?

14.(10 pts) If matrix $A \in M(n, n)$ is diagonalizable with eigenvalues are either 1 or-1. Show that $A^2 = I$.

15. (13 pts). Let $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.

- **(3pts).** Does the system $AX = B$ have a solution X? Why?

- **(10pts).** Find a leat square solution to the system $AX = B$.

12 (12pts). Determine numbers a, b, c and d such that the matrix A is orthogonal:

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} & a \\ \frac{1}{2} & \frac{-1}{2} & 0 & b \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} & c \\ \frac{1}{2} & \frac{-1}{2} & 0 & d \end{bmatrix}$$