NAME ____________________ Recitation Instructor ____________________

Signature ____________________ Recitation Time ____________________

Elementary Differential Equations
Math 240, Spring 2007
First Examination, February 13, 2007

page	1	2	3	4	5	6	7	8	9	10	total
points											

Show all your work in the space under each question. Please write legibly and organize your solutions in a logical and coherent form; answers which are illegible or confusing will not receive credit. Each problem is worth 10 points.

1. Find the general solution.

$$y' = y \tan x + \sin x$$

2. Find the general solution.

$$\sin(x+y)dx + (2y + \sin(x+y))dy = 0$$

3. Find the general solution.

$$x\frac{dy}{dx} + 6y = 3xy^{\frac{4}{3}}$$

4. Use the Euler method with stepsize $h = .5$ to approximate $y(1)$ for the initial value problem.

$$y' = x^2 + y \qquad y(0) = 2$$

5. Find the general solution.

$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}$$

6. Consider the initial value problem

$$\frac{dP}{dt} = (P-1)(P-2) \qquad P(0) = 3.$$

Determine (by solving the equation) the behavior of the solution as t starts at 0 and increases.

7. Find the general solution.

$$\frac{dy}{dx} = 6x(y-1)^{\frac{2}{3}}$$

8. The number of prairie dogs (measured in hundreds) in a colony is described by the equation

$$\frac{dP}{dt} = -P(P-1)(P-20).$$

(2 pts.) (a) Find the critical points of this equation.

(4 pts.) (b) Determine the intervals on which P is increasing and decreasing. Draw a graph of P with several different initial values. Your graph must be well drawn and readable.

(4 pts.) (c) Determine if each critical point you found in (a) is stable or unstable. Identify the threshold and carrying capacity.

9. Use the improved Euler method with stepsize 1 to estimate $y(3)$ for the initial value problem:

$$y' = x + 1 \qquad y(1) = 0$$

10. Match the slope fields to the equations.

(a) $y' = .2x$
(b) $y' = xy$
(c) $y' = y$
(d) $y' = x$
(e) $y' = x^2 - y^2$

This is the slope field of equation _____.

This is the slope field of equation _____.

This is the slope field of equation _____.

This is the slope field of equation _____.

This is the slope field of equation _____.