Math 240: Elementary Differential Equations

Exam 3: Summer 2009

Name: _____

Instructor: _____

This is a closed book exam. No books, notes or calculators are allowed. Laplace transforms table is provided on the last page of this exam. You must show all your work to receive credit. Please note that some problems require using a certain method. If you use another method, you will not received credit.

(10) 1. Solve the system

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = -4x - 3y$$

(15) 2. Solve the system using Laplace transforms

$$\frac{dx}{dt} = 2x + y \qquad x(0) = 0$$
$$\frac{dy}{dt} = -4x - 3y \qquad y(0) = 3$$

(10) 3. Solve, using Laplace transforms, the initial value problem

$$x''' - x'' + x' - x = 0$$

$$x(0) = 0, \quad x'(0) = 1, \quad x''(0) = 1$$

(15) 4. (a) Rewrite the following equation (together with initial conditions) as a first order system:

$$x'' - 2tx' + x = 0, \qquad x(0) = 0, \qquad x'(0) = 1$$

(b) Use Euler's method with step size $h = 0.5$ to approximate $x(1)$ and $y(1)$ for this system.

(10) 5. Use the improved Euler's method with step size $h = 1$ to approximate $y(2)$ for the following initial value problem:

$$\frac{dy}{dx} = xy + x + 2, \quad y(1) = 0$$

(10) 6. Solve the initial value problem:

$$x'' + 9x = 2\delta(t) + u(t-3), \quad x(0) = 0, \quad x'(0) = 0$$

(10) 7. Solve, using Laplace transforms, the initial value problem. You will need to express a part of you solution as a convolution integral.

$$x'' + 2x' + 2x = e^{\cos t}, \quad x(0) = 0, \quad x'(0) = 1$$

(10) 8.(a) Find the Laplace transform of the function:

$$f(t) = t^3 - 7 + \int_0^t \sin(t-\tau)\cos(2\tau)d\tau$$

(b) Find the inverse Laplace transform of the function:

$$\frac{s}{s^4 + 2s^2 + 1}$$

(10) 9.(a) Compute $\Gamma(4)$

(b) If $\alpha = \Gamma(\frac{1}{4})$, express $\Gamma(-\frac{7}{4})$ as a function of α.