Math 240: Elementary Differential Equations

Exam 3: Summer 2008

July 18, 2008

Name.......................

Instructor.................

This is a closed book exam. You can use a calculator and 8.5" $\times$ 11" sheet of hand written note (both sides).Detail work must be shown for the full credit.

(10)Problem.1. Approximate $y(0.2)$ using Euler's method with a step size of $h = .1$ where y is the solution to the initial value problem

$$\frac{dy}{dx} = 2x + y, \quad y(0) = 1.$$

(10)Problem 2(a). Find the inverse Laplace Transform of

$$\frac{2s-1}{s^2-6s+13}$$

(b). Evaluate $\Gamma(-\frac{7}{3})$ in terms of $\Gamma(\frac{2}{3})$.

(12) Problem 3. Solve the initial value problem using the Laplace transform

$$y'' + 2y' + 2y = e^t, \quad y(0) = 1, y'(0) = 0.$$

(12)Problem 4. Solve the initial value problem by using the Laplace Transform

$$x'' + 6x' + 34x = \delta(t-1), \quad x(0) = 0, x'(0) = 1.$$

(12)Problem 5. Solve the following initial value problem by using Laplace Transform

$$x'' - x' - 6x = \sqrt{t}, \quad x(0) = 1, x'(0) = 1.$$

(12)Problem 6. Solve the system of equations by using the Laplace Transform

$$\begin{aligned}\frac{dx}{dt} &= 3x - 4y \qquad x(0) = 1\\ \frac{dy}{dt} &= 2x - y \qquad y(0) = -2\end{aligned}$$

(12)**Problem 7.** Consider the following differential equation :

$$mx'' + cx' + kx = \delta(t-1), \quad x(0) = 1, x'(0) = 1.$$

Find the location of the poles if

(a)$m > 0, c > 0, k > 0$ and $c^2 - 4km < 0$.

(b) $m > 0, c > 0, k > 0$ and $c^2 - 4km > 0$.

(c) $m > 0, c > 0, k > 0$ and $c^2 - 4km = 0$.

(d) $m > 0, k > 0, c = 0$.