Math 240: Elementary Differential Equations

Exam 2: Summer 2009

Name: _____

Instructor: _____

This is a closed book exam. No books, notes or calculators are allowed. You must show all your work to receive credit.

(10) 1. Find the general solution of the differential equation

$$y^{(4)} + 4y'' + 4y = 0$$

(10) 2. Find the general solution of the differential equation

$$y''' - y = 2\sin x$$

(15) 3. Solve the initial value problem

$$y'' - 2y' + y = 4e^{-x} + x^2$$

$$y(0) = 8, \quad y'(0) = 4$$

(15) 4. Find the general solution of the differential equation (you must compute all integrals for a full credit)

$$y'' + y = \frac{1}{\sin x} + x$$

(15) 5. A mass of $\frac{1}{2}kg$ is attached to a spring with a spring constant $2kg/sec^2$. The spring is then pulled down an additional $1m$ and released.

(a) For what (positive) values of c (damping constant) is this underdamped? Overdamped? Critically damped?

(b) Find the equation of resulting motion if $c = \frac{5}{2}kg/sec$

(15) 6. A mass of $1kg$ is attached to an undamped spring with a spring constant $4kg/sec^2$. The mass is subject to an external force of $4\cos(2t)$ Newtons. There is no initial displacement or velocity. Find the equation of the resulting motion.

(10) 7. A mass of $100g$ is attached to an undamped spring, which causes the spring to stretch $98cm$. The mass then was pulled down an additional $30cm$ and given a downward velocity of $4m/sec$. Find the amplitude of the resulting motion (use $g = 9.8m/sec^2$).

(10) 8. Match the graphs to the equations. Each graph ranges from 0 to 6π on the horizontal scale and from -10 to 10 on the vertical scale.

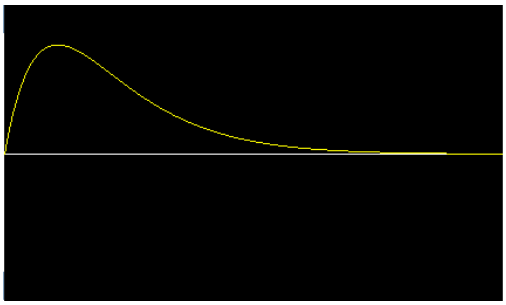
(a) $5x'' + x' + 5x = 0 \qquad x(0) = 10,\ x'(0) = 0$

(b) $x'' + 5x' + x = 0 \qquad x(0) = 10,\ x'(0) = 0$

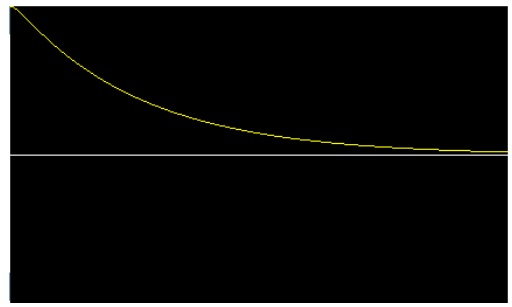
(c) $5x'' + 5x = 4\cos t \qquad x(0) = 0,\ x'(0) = 0$

(d) $x'' + 0.5x' + 2x = 10 \qquad x(0) = 0,\ x'(0) = 0$

(e) $x'' + x' + 0.25x = 0 \qquad x(0) = 0,\ x'(0) = 10$



This is the graph of equation ___



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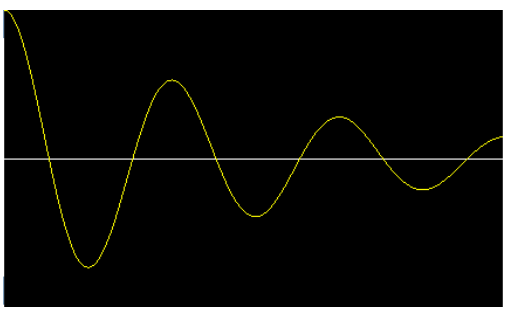

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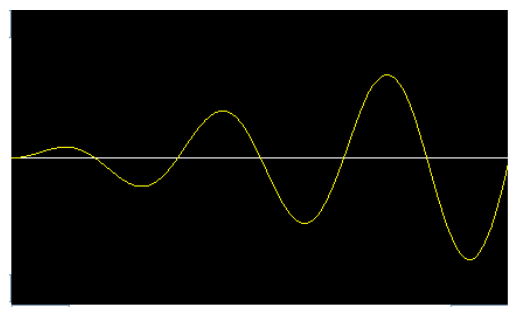

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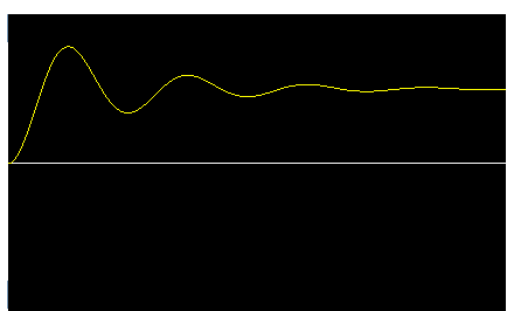

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