Math 240: Elementary Differential Equations

Exam 1: Summer 2008

June 20, 2008

Name.....

Instructor.....

This is a closed book exam. You can use a calculator and 8.5" $\times$ 11" sheet of hand written note (both sides).Detail work must be shown for the full credit.

(10)Problem.1. Find the general solution of the differential equation

$$(x^3 + \frac{y}{x})dx + (y^2 + \ln x)dy = 0, \quad x > 0.$$

(10)Problem 2. Solve the initial value problem :

$$\frac{dy}{dx} + \frac{4}{x}y = x^3y^2, y(1) = -1, x > 0.$$

(10) Problem 3. Find all solutions to the equation

$$\frac{dy}{dx} = e^{-y}(2x - 4).$$

(10)Problem 4. Solve the equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}, y(\sqrt{2}) = 1.$$

(20)Problem 5. Suppose

$$\frac{dP}{dt} = -P^3 + 7P^2 - 10P.$$

(a) Find the equilibrium points of the equation. Sketch the several solution curves indicating the threshold. Also classify the equilibrium points as stable or unstable.

(b) If $P(0) = 6$ find the limits

$$\lim_{t\to\infty} P(t) \quad \text{and} \quad \lim_{t\to-\infty} P(t).$$

(c). Suppose $P(t)$ represents population of fish in a lake in thousands.What is the fate of the population of the fish in the lake if the current population of the fish is 1500?

(d). How many fish should be in the lake in order to prevent the extinction ?

(10)**Problem 6 (a).** Express the following in the form of $a + bi$.

$$\frac{2e^{-i\frac{3\pi}{4}}}{2-i}$$

(b). Express $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ into polar form.

(10)**Problem 7(a).** Write the following in the form of $A\cos(\omega x - \phi)$

$$3cos(2x) + 3\sqrt{3}sin(2x).$$

(b). Express $6\cos(2x - \frac{\pi}{3})$ in the form of $a\cos(\omega x) + b\sin(\omega x)$.