Math 240: Elementary Differential Equations

Exam 1: Summer 2009

Name: _____

Instructor: _____

This is a closed book exam. No books, notes or calculators are allowed. You must show all your work to receive credit.

(10) 1. Find all solutions of the differential equation (implicit form is accepted)

$$1 + 2\sqrt{y}e^x + (\frac{e^x}{\sqrt{y}} - 1)\frac{dy}{dx} = 0$$

(10) 2. Find all solutions to the equation

$$\frac{dy}{dx} + 5y = y^2$$

(10) 3. Find all solutions to the equation

$$\frac{dy}{dx} = \frac{y}{1+x^2}$$

(15) 4. Solve the initial value problem

$$\frac{dy}{dx} = \frac{y^2 + 4x^2}{4xy}, \qquad y(1) = 2$$

(10) 5. Find all solutions to the equation

$$x\frac{dy}{dx} + 2y = \frac{1}{x}$$

(10) 6. Write the expression in the form $a + bi$, where a and b are real numbers:

(a) $\dfrac{2-5i}{1-3i}$

(b) $(1-i)^{20}$ (Hint: use exponential form)

(10) 7.

(a) Express in the form $A\cos(\omega x - \phi)$:

$$\sin(3x) - \sqrt{3}\cos(3x)$$

(b) Express in the form $a\cos(\omega x) + b\sin(\omega x)$:

$$\sqrt{2}\sin(2x + \frac{\pi}{4})$$

(15) 8. Suppose that population of bisons in Konza Prairie satisfies the differential equation

$$\frac{dP}{dt} = -0.1 * P(P - 300)(P - 100)$$

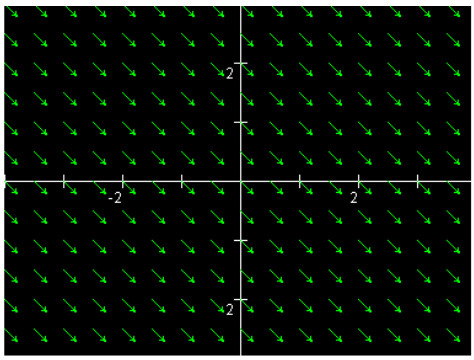
(a) Find the equilibrium points of the equation. Sketch several trajectories, including equilibrium solutions.

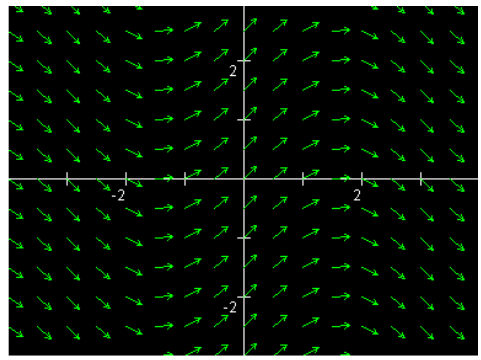
(b) Assume current population is equal to 85 bisons. Are they in danger of extinction? How many bisons do you need to buy to prevent extinction? Explain.

(c) Assume after the population had reached 275 bisons, additional 40 bisons were donated to Konza Prairie. Is that wise to accept the donation? Explain why/why not.

(10) 9. Match the slope fields to the equations.

(a) $y' = -1$ (b) $y' = xy$ (c) $y' = -xy$ (d) $y' = y^2 - 1$ (e) $y' = cos(x)$





This is the slope field of equation ___

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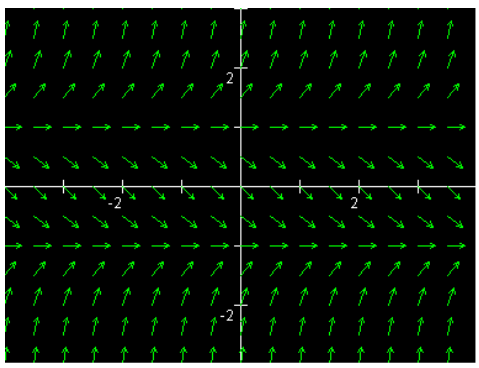

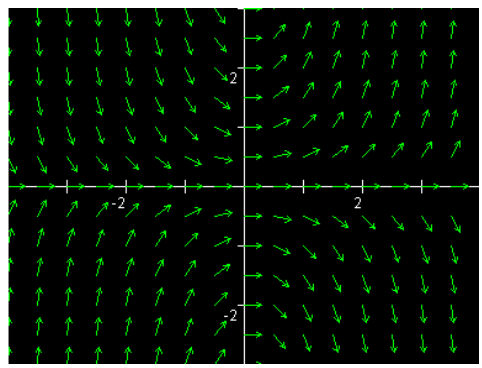

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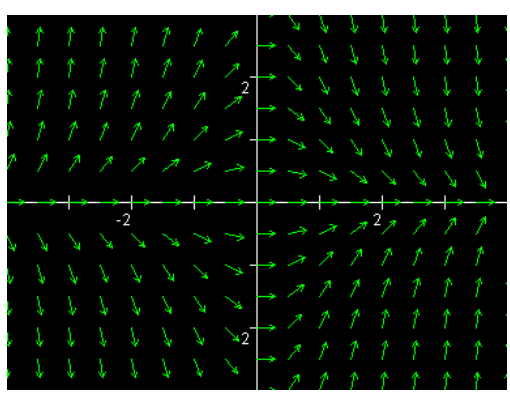

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