Name _____ Recitation Time _____

Signature _____ Recitation Instructor _____

Differential Equations - MATH 240

Third Examination, November 14, 2006

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points											

Show all your work in the space provided under each question.

1. Find the general solution to the system of equations.

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 4x + y$$

2. (a) What is $\Gamma(7)$?

(b) Given that $\Gamma(.5) = \sqrt{\pi}$, find $\Gamma(-\frac{7}{2})$.

3. Use the Euler method with stepsize $h = .5$ to estimate $x(2)$ and $y(2)$ for the initial value problem

$$\begin{aligned} x' &= 2x + t \qquad & x(1) &= 0 \\ y' &= xy \qquad & y(1) &= 2 \end{aligned}$$

4. Solve, using Laplace transforms, the initial value problem

$$y'' - 2y' + 2y = 0, \qquad y(0) = 1, \quad y'(0) = 0.$$

Although it is possible to solve this with other methods, you will receive credit only if you use Laplace transforms.

5. A mass-spring system has a mass of $1kg$, a spring constant of $2\frac{kg}{sec^2}$ and a damping constant of $2\frac{kg}{sec}$. The system is set into motion with a unit impulse at time $t = 1$, but is given no initial displacement or velocity. Find $x(t)$ which gives the position of the mass at time t.

6. Solve using Laplace transforms.

$$y^{(4)} - y = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 0$$

7. Solve. You may need to express part of your solution as a convolution integral.

$$x'' + 2x' + x = \sqrt{t}, \qquad x(0) = 0, \quad x'(0) = 1$$

8. Consider the differential equation

$$x'' + 2x' + 3x = \delta(t), \qquad x(0) = 0, \quad x'(0) = 1$$

(a)What is the location of the poles of $\mathcal{L}\{x\}$ in the complex plane?

(b) What is the behavior of the solution $x(t)$ as $t \to \infty$?

9. Solve the system of equations using Laplace transforms.

$$x' = -4x - 3y \qquad x(0) = 0$$

$$y' = 6x + 5y \qquad y(0) = 1$$

10. (a) Write the second order differential equation as a system of first order equations.

$$x'' + 2x' + 6tx = \sin t$$

(b) Explain why it is not possible to compute $\Gamma(-1)$ using the integral formula

$$\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt.$$