NAME _____ Recitation Instructor _____

Signature _____ Recitation Time _____

Elementary Differential Equations
Math 240, Fall 2006
Second Examination, October 17, 2006

page	1	2	3	4	5	6	7	8	9	10	total
points											

Show all your work in the space under each question. Please write legibly and organize your solutions in a logical and coherent form; answers which are illegible or confusing will not receive credit. Each problem is worth 10 points.

1. Solve the initial value problem

$$y'' + 2y' - 24y = 0 \qquad y(0) = 0,\ y'(0) = 1.$$

2. Find the general solution.

$$4y'' - 4y' + y = 0$$

3. Find the general solution.

$$y'''' + 2y'' + y = 0$$

4. Rewrite in the form $\cos(\omega x - b)$.

$$\cos 2x + \sqrt{3}\sin 2x$$

5. Find the general solution.

$$y'' + y' - 2y = xe^x$$

6. A mass of $2kg$ is attached to an undamped spring which causes the spring to stretch $19.6m$. This is attached to a damping mechanism with damping constant of $3\frac{kg}{sec}$. After coming to equilibrium, the mass is set into motion with no initial displacement and an initial velocity $1\frac{m}{sec}$ downward. There is no forcing. Find the function $x(t)$ which gives the displacement of the mass at all future times t.

7. Find the general solution.

$$y'' - 2y' + 2y = e^x \sec x$$

8. A mass-spring-damping system has a mass of $1kg$, a damping constant of $7\frac{kg}{sec}$ and a spring constant of $12\frac{kg}{sec^2}$. After coming to equilibrium, the mass is set into motion by pulling it down $1m$ and then releasing it. Find the energy of the system as a function of the time t.

9. A mass of $4kg$ is attached to an undamped spring with spring constant $4\frac{kg}{sec^2}$. The mass is allowed to come to equilibrium. Beginning at time $t = 0$ the mass is subject to an external force of $\sin 2t\ N$. There is no initial displacement or velocity. Find the function $x(t)$ which gives the displacement of the mass from equilibrium at time t.

10. Match the graphs to the equations. Each graph ranges from 0 to 20 on the horizontal scale and from -10 to 10 on the vertical scale.

(a) $x'' + x' + 4x = 50\cos(4t) \quad x(0) = 0,\ x'(0) = 0$

(b) $x'' + .5x' + 50x = 0 \quad x(0) = 10,\ x'(0) = 0$

(c) $x'' + x = 0 \quad x(0) = 0,\ x'(0) = 10$

(d) $x'' + 16x = 4\cos(4t) \quad x(0) = 0,\ x'(0) = 0$

(e) $x'' + 4x' + 3x = 0 \quad x(0) = 10,\ x'(0) = 0$

This is the graph of equation _____.

This is the graph of equation _____.

This is the graph of equation _____.

This is the graph of equation _____.

This is the graph of equation _____.