Elementary Differential Equations-Math 240
Final Exam–Summer 2005
July 29,2005

Name:

Instructor:

Please show all work.Answers with no work will receive non credit.No calculatrs or notes are allowed on this exam

The formula for Variation of Parameters is:

$$y_p = -y_1(x)\int \frac{y_2(t)g(t)}{W(y_1,y_2)(t)}dt + y_2(x)\int \frac{y_1(t)g(t)}{W(y_1,y_2)(t)}dt$$

1. (10 pts) Solve the following differential equation:

$$\frac{dy}{dx} = \frac{5x-2y}{y+2x} \quad y(0)=1$$

2. (10 pts) Convert $-2\sqrt{2}cos(7x) + 2\sqrt{2}sin(7x)$ to $Acos(\omega x - \theta))$

3. (10 pts) Solve the following differential equation:

$$y' - \frac{3}{x}y = e^x x^4$$

4. (10 pts) Find the Taylor series and Radius of convergence for the function

$$f(x) = \frac{x}{1-3x}$$

5. (10 pts) Find and classify the equiblibrium points of

$$\frac{dp}{dt} = -(p-1.5)(p-4)p$$

and label the threshold and carrying capacity of this population model.

6. (10 pts) Solve the following Initial Value Problem

$$x'' + 7x' - 8x = \delta(t) \quad y(0) = 1; y'(0) = 2$$

7. (10 pts) Find the solution to the Boundary Value Problem

$$x'' - 6x' + 10x = sin(t) \quad x(0) = 3, x(\pi) = -3$$

8. (10 pts) Solve the following system of differential equations

$$\frac{dx}{dt} = x + y \quad x(0) = 2$$

$$\frac{dy}{dt} = -3x + 2y \quad y(0) = 1$$

9. (15 pts)A mass of $5kg$ is attached to spring with a spring constant $10kg/sec^2$ The spring has a damping constant of $15kg/sec$.The spring is under an external force of $25sin(4t)$ newtons. Find the equation of the resulting motion and solve it, and state the steady state solution.

10. (10 pts) Find and classify the singular points of

$$(2x+1)^4(x-1)y'' - 4(2x+1)^3y' + 5y = 0$$

11. (15 pts) Solve the following Differential equation

$$x^2y'' - xy' - 3y = e^{-x}$$

12. (15 pts) Find the power series solution about $x = 0$ for the following differential equation(find the first three non-zero terms)

$$y'' + xy' - 3xy = 0 \quad y(0) = 1, y'(0) = 2$$

13. (15 pts) Find the first three non-zero terms of the Series solution about $x = 0$ that corresponding to the larger root of the indicial equation of

$$2x^2y'' - 3xy' + 6y = 0$$