Name _____ Recitation Time_____

Signature _____ Recitation Instructor_____

Differential Equations - MATH 240

Final Exam, December 11, 2006

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points								
page	9	10	11	12	13	14	15	total
points								

Show all your work in the space provided under each question. Each problem is worth 10 points.

1. Find the general solution to $(1 + e^x y)dx + e^x dy = 0$.

2. Use the Euler method with stepsize .5 to estimate $y(1)$ for the initial value problem.

$$y' = xy + y^2 \qquad y(0) = 1$$

3. The population $P(t)$ (measured in thousands) of catfish in a lake is described by the equation

$$\frac{dP}{dt} = -P(P-10)(P-1)$$

(a) Find the equilibrium points.

(b) Draw several trajectories. Include in your graph the critical points and label the threshold and carrying capacity.

(c) Classify each critical point as stable or unstable.

(d) In order to increase the number of fish in the lake, it is decided to stock the lake so as to bring the number of fish up to 12,000. Is this a good idea? Explain your answer.

4. Find the general solution: $y' - y = xy^2$.

5. Solve the initial value problem

$$x'' + 2x' + 2x = \delta(t - \pi) + h(t) \qquad x(0) = 0, \quad x'(0) = 0.$$

You will need to express a part of your solution as a convolution integral involving the function $h(t)$.

6. Find the general solution.

$$x^2y'' + xy' - y = x^4 \qquad \text{(assume } \ x > 0\text{)}.$$

7. Use the improved Euler method with stepsize $h = 1$ to estimate $x(1)$ and $y(1)$ for the initial value problem

$$\begin{aligned} x' &= x + y \qquad & x(0) &= 0 \\ y' &= tx \qquad & y(0) &= 1. \end{aligned}$$

8. Solve $y'' - xy = 0$, $y(0) = 0$, $y'(0) = 1$ using power series. Find the recurrence relation and the first three nonzero terms of the series solution.

9. Consider the differential equation

$$(x-2)(x-3)^2y'' + (x-3)y' + (x-2)y = 0.$$

(a) Find the ordinary points and the singular points of this differential equation.

(b) Classify each singular point you found in (a) as a regular singular point or an irregular singular point.

(c) It is desired to solve this equation using a power series centered at $x = 5$. Give a lower bound for the radius of convergence of such a power series solution.

10. (a) Express $\Gamma(\frac{7}{3})$ in terms of $\Gamma(\frac{1}{3})$.

(b) Write the second order differential equation as a first order system.

$$x'' + tx' + 3x = \sin t$$

11. A mass of $2\,kg$ is attached to a spring which causes the spring to stretch $4.9\,m$. To the mass is attached a damping mechanism with damping constant $6\,\frac{kg}{sec}$. In addition, this system is subjected to a variable force of $10\cos t\,N$. Find $x(t)$, the function which gives the position of the mass at time t. Use $g = 9.8\,\frac{m}{sec^2}$. (Note that no initial conditions are given so that your solution will involve constants.)

12. Consider the differential equation

$$x'' + cx' + 4x = F_0 \cos \omega t$$

which describes a mass-spring-damping system. Here c and F_0 are nonnegative constants.

(a) Suppose $F_0 = 0$. For which values of c is the system underdamped? For which values of c is the system critically damped?

(b) Suppose $F_0 = 1$, $c = 0$. For which value of ω is there resonance?

13. Find the Frobenius series solution corresponding to the larger root of the indicial equation of the differential equation

$$2x^2y'' - xy' + xy = 0.$$

Give at least three nonzero terms in the series.

14.Match the slope fields to the equations.

(a) $y' = x$
(b) $y' = x - y$
(c) $y' = 1$
(d) $y' = y$
(e) $y' = .2x$

This is the slope field of equation ____.

This is the slope field of equation ____.

This is the slope field of equation ____.

This is the slope field of equation ____.

This is the slope field of equation ____.

15. Match the graphs to the equations. Each graph ranges from 0 to 20 on the horizontal scale and from -10 to 10 on the vertical scale.

(a) $x'' + x = 0 \quad x(0) = 0,\ x'(0) = 9$

(b) $x'' + x' + x = 10\cos t \quad x(0) = 0,\ x'(0) = 0$

(c) $x'' + 400x = 200\cos 18t \quad x(0) = 0,\ x'(0) = 0$

(d) $x'' + 10x' + 3x = 0 \quad x(0) = 0,\ x'(0) = 100$

(e) $5x'' + x' + 10x = 0 \quad x(0) = 10,\ x'(0) = 0$

This is the graph of equation ____. This is the graph of equation ____.

This is the graph of equation ____. This is the graph of equation ____.

This is the graph of equation ____.