CALCULUS III NAME _____

EXAM III Rec. Instr. _____

SPRING 1997 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(10) 1. Find the equation of the plane which is tangent to the surface $x^3y + y^3z + z^3x = 5$ at the point $(x, y, z) = (1, -1, 2)$.

(20) 2. Let $f(x, y) = x^2 e^{xy}$.

Find

a) the gradient vector field of f.

b) the directional derivative of $f(x, y)$ at $(x, y) = (1, 2)$ in the direction of the vector $\vec{a} = -\vec{i} + \vec{j}$.

c) a unit vector which points in the direction which gives the largest directional derivative of f at $(1, 2)$.

d) the value of the largest directional derivative of f at $(1, 2)$.

(20) 3. Find and classify the critical points for the function

$$f(x, y) = y(x^2 + 2x - 3) - \frac{y^2}{2}$$

(10) 4. *Use the method of Lagrange multipliers* to find the maximum value and the minimum value of $f(x, y) = xy$ on the ellipse $x^2 + 3y^2 = 12$.

(15) 5. Find the volume of the 3-dimensional region which is under the surface $z = 3 + x$ and above the region of the xy-plane which is enclosed by the curves $y = x^2$ and $y = 4$.

(10) 6. Evaluate the iterated integral below *by first reversing the order of integration.*

$$\int_0^4 \int_{\sqrt{x}}^2 9\cos(y^3)\,dy\,dx$$

(15) 7. Find the volume of the 3-dimensional region which is bounded by the paraboloids $z = 6 - x^2 - y^2$ and $z = x^2 + y^2$. Use polar coordinates to evaluate the double integral.