CALCULUS III NAME _____

EXAM III Rec. Instr. _____

SPRING 2000 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. Find the volume of the 3-dimensional region which is inside the sphere $x^2 + y^2 + z^2 = 2$ and above the paraboloid $z = x^2 + y^2$.

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(15) 2. A mass distribution occupies the region in the 1st octant enclosed by the surfaces $z = x$, $z = 0$, $y = 1$, $y = 1 + x^2$ and $x = 2$. If the mass density function is $\delta = \dfrac{4}{1 + x^2}$ units of mass/unit volume, calculate the total mass.

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(15) 3. Use a triple integral in spherical coordinates to calculate the total mass of a mass distribution which lies above the cone $\phi = \cos^{-1}\left(\frac{4}{5}\right)$ and under the plane $z = 2$, given that the mass density function is $\delta = z^2$ units of mass/unit volume.

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(10) 4. Find the area of that part of the plane $2x + y + 4z = 8$ which is in the 1st octant and is between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

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(15) 5. Calculate the area of the parametrized surface $x = t\,,\ y = t^2\,,\ z = ts^3\,,$ $0 \le t \le 3\,,\ 0 \le s \le 2\,.$

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(20) 6. Evaluate the line integral

$$\int_C (1+xy)dx + \left(\frac{1}{1+x^2}\right) dy$$

for each of the following curves from $(0,0)$ to $(2,4)$.

a) C is the parabola $y = x^2$ from $(0,0)$ to $(2,4)$.

b) C consists of the line segment from $(0,0)$ to $(2,0)$ followed by the line segment from $(2,0)$ to $(2,4)$.

c) C consists of the line segment from $(0,0)$ to $(0,4)$ followed by the line segment from $(0,4)$ to $(2,4)$.

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(10) 7. Show that the force field

$$\vec{F} = -\frac{(4+y^2)}{(1+x)^2}\vec{i} + \left(2y + \left(\frac{2y}{1+x}\right)\right)\vec{j}$$

is conservative in the region $x > -1$ by finding a potential function for it. Now for each of the curves a), b) and c) of problem 6 calculate the work done by $\vec{F}$ as it acts on an object which moves along the curve.