CALCULUS III NAME _____

EXAM III Rec. Instr. _____

FALL 1999 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. A mass distribution occupies the region in the first octant which is enclosed by the surfaces $y = x$, $x = 1$, $y = 0$, $z = 1 + x$, $z = 0$. If the mass density function is $\delta = x^2 + y^2$ units of mass/unit volume, calculate the total mass.

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(15) 2. Evaluate the surface integral $\int\int_S (z^2 - y^2)\, dS$ where S is the parametrized surface $x = u^2 + v^2\,,\ y = u - v\,,\ z = u + v\,,\ 0 \le u \le 1\,,\ 0 \le v \le 1\,.$

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(15) 3. Evaluate the surface integral, $\int\int_S (\vec{F} \cdot \vec{n})\, dS$ where $\vec{n}$ is the upward pointing unit normal vector to the surface S, $\vec{F} = x\vec{i} + y\vec{j} + 5z\vec{k}$, and S is that part of $z = x^2 + y^2$ which is under the plane $z = 4$.

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(15) 4. The force field $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ as the object moves from $(1,0)$ to $(3,1)$ along the following path:

first, along $x^2 + y^2 = 1$ in the first quadrant from $(1,0)$ to $(0,1)$.

second, along the line segment from $(0,1)$ to $(3,1)$.

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(15) 5. Use Green's theorem to evaluate

$$\int_C (x^2 + y^2)\, dx + (x^2 + y^2 + 2xy)\, dy$$

where C consists of: the x-axis from $(0,0)$ to $(2,0)$, the circle $x^2+y^2 = 4$ in the first quadrant from $(2,0)$ to $(0,2)$, and the y-axis from $(0,2)$ to $(0,0)$.

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(15) 6. Use the divergence theorem to calculate the outward flux integral $\int\int_S (\vec{F}\cdot\vec{n})\, dS$ where $\vec{F} = xz\vec{i}+yz\vec{j}+xy\vec{k}$ and S consists of the hemisphere $x^2+y^2+z^2 = 4$, $z \geq 0$ together with the disk, $x^2+y^2 \leq 4$ in the xy-plane. Use spherical coordinates to do the triple integral.

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(10) 7. Show that the force field $\vec{F} = \left(\frac{1}{y} + 2x\right)\vec{i} + \left(3y^2 - \frac{x}{y^2}\right)\vec{j}$ is conservative in the region $y \neq 0$ by finding a potential function for it. Now use this potential function to calculate the work done by $\vec{F}$ as it acts on an object which moves from $(x, y) = (2, 1)$ to $(x, y) = (1, 3)$ along any curve in the upper half plane.