CALCULUS III NAME _____

EXAM III Rec. Instr. _____

FALL 2002 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1) A mass distribution occupies the 3D region which is enclosed by the surfaces $z = x^2 + y^2$, $z = 0$ and $x^2 + y^2 = 4$. If the mass density function is $\delta = x^2 + y^2 + z^2$ units of mass/unit volume, calculate the total mass.

Name: _____

Recitation Time: _____

(20) 2) A mass distribution occupies the 3D region which is above the surface $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 9$. If the mass density function is $\delta = \dfrac{1}{z}$ units of mass/unit volume use a <u>triple integral in spherical coordinates</u> to calculate the total mass.

Name: _____

Recitation Time: _____

(15) 3) Calculate the surface area of the parametrized surface $x = 2t^2$, $y = t$, $z = st$, $0 \leq s \leq 3$, $0 \leq t \leq 2$.

Name: _____

Recitation Time: _____

(20) 4) The force field $\vec{F} = xy\vec{i} + x^2\vec{j}$ acts on an object as it moves in the plane. Calculate the work done by $\vec{F}$ for each of the following motions.

a) The object moves from $(0, 0)$ to $(2, 1)$ along the line $x = 2y$.

b) The object moves from $(0, 0)$ to $(2, 1)$ by first going along $x = y^2$ from $(0, 0)$ to $(1, 1)$ and then along the line segment from $(1, 1)$ to $(2, 1)$.

Name: _____

Recitation Time: _____

(10) 5) Show that the force field

$$\vec{F} = \left(y + 2x + \frac{y}{x^2}\right)\vec{i} + \left(x + 1 - \frac{1}{x}\right)\vec{j}$$

is conservative in the region $x > 0$ by finding a potential function for it. Now use this potential function to find the work done by $\vec{F}$ as it acts on an object which moves along a path in the right half plane from $(1, 1)$ to $(2, 4)$.

Name: _____

Recitation Time: _____

(15) 6) Use Green's theorem to evaluate the line integral

$$\int_C (y^2 + ye^{xy})dx + (4xy + xe^{xy})dy$$

where C is the closed path made up of the sides of the triangle with vertices $(0, 0)$, $(1, 1)$ and $(2, 0)$ directed counterclockwise.