CALCULUS III NAME _____

EXAM III Rec. Instr. _____

FALL 2000 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1) A mass distribution occupies the region in 3-space enclosed by the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 2$. Its mass density function is $\delta(x, y, z) = x^2 + y^2 + z^2$. Use a triple integral in **spherical coordinates** to calculate the total mass.

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(15) 2) Find the surface area of the parametrized surface $x = t$, $y = t^2$, $z = ts^3$, $0 \leq t \leq 2$ and $1 \leq s \leq 3$.

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(15) 3) Evaluate the line integral $\int_C xy\,dx + x\,dy$ where C consists of the line segment from $(2,-4)$ to $(2,0)$ followed by the arc of the circle $x^2+y^2=4$ from $(2,0)$ to $(0,2)$ which is in the first quadrant.

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(10) 4) Show that the Force field

$$\vec{F} = \left(2 + \frac{2x}{1+y}\right)\vec{i} + \left(2y - \frac{x^2}{(1+y)^2}\right)\vec{j}$$

is conservative in the region $y > -1$ by finding a potential function for it. Now calculate the work done by $\vec{F}$ as it acts on an object which moves from $(1, 0)$ to $(9, 3)$ along any piecewise smooth curve in the region $y > -1$.

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(15) 5) Use Green's theorem to evaluate

$$\int_C (xy + y^3 \cos x)dx + (3y^2 \sin x + x^2)dy$$

where C is the closed curve consisting of the sides of the triangle having vertices $(0,0)$, $(2,0)$, $(0,2)$, directed counterclockwise.

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(15) 6) Use Green's theorem in vector form to calculate the outward flux $\int_C \vec{F} \cdot \vec{n}\, ds$ where $\vec{F} = xy^2\,\vec{i} + x^2y\,\vec{j}$ and C is the circle $x^2 + y^2 = 4$.

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(15) 7) Evaluate the surface integral $\int\int_S 12xy\,dS$ where S is that part of $z = \sqrt{25 - x^2 - y^2}$ which lies above the square $0 \le x \le 1$, $0 \le y \le 1$.