CALCULUS III NAME _____

EXAM II Rec. Instr. _____

SPRING 1999 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. A quantity Q depends upon x and y according to $Q = x^2 e^{-xy^2}$. Suppose x and y are changing with time t and at a certain instant you know that $x = 3$, $y = 1$, $\dfrac{dx}{dt} = 2$ and $\dfrac{dy}{dt} = \dfrac{1}{2}$. Use the chain rule to calculate $\dfrac{dQ}{dt}$ at this instant.

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(15) 2. Find the equation of the tangent planes to the following surfaces at the specified points.

a) To the graph of $f(x, y) = x\sqrt{y^2 + x}$ at $(3, 1, 6)$.

b) To $x^3y + x^2z + y^2z^3 = 11$ at $(1, 1, 2)$.

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(20) 3. Let $f(x,y) = x^3y + 2xy^2 - 6x + 2y + 3$.

a) Find the gradient vector field of f.

$\nabla f =$

b) Suppose you are moving along a straight line through the point $P(1,2)$ toward the point $Q(4,1)$. Calculate the instantaneous rate of change of $f(x,y)$ per unit distance that you will observe at $P(1,2)$.

c) In which direction should you go when you are at $P(1,2)$ if you want $f(x,y)$ to increase as rapidly as possible?

d) What is the value of the largest possible rate of change for $f(x,y)$ at $P(1,2)$?

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(15) 4. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y, z) = x + 2y + z^2$ on the ellipsoid $x^2 + y^2 + 2z^2 = 20$.

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(20) 5. Let $f(x,y) = x^3y^2 + 2xy - 2x$.

a) Calculate all 1st and 2nd partial derivatives of $f(x,y)$.

b) Find all critical points of $f(x,y)$. These are points where the tangent plane is horizontal.

c) Use the 2nd derivative test to classify the critical points of $f(x,y)$.

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(15) 6. Carefully sketch the following surfaces in 3-space.

a) $z = 9 - x^2 - y^2$

b) $z^2 = 9 - x^2 - y^2$

c) $z = 9 - x^2$

d) $2x + 3y + z = 6$