CALCULUS III NAME _____

EXAM II Rec. Instr. _____

SPRING 1997 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1. An object is moving in 3-space according to the parametric equations $x = \cos t$, $y = \sin t$ and $z = 4t^2$. Find as functions of time t

a) unit tangent vector $\vec{T}$.

b) tangential component of acceleration a_T.

Now at $t = \dfrac{\pi}{2}$ seconds find

c) curvature κ.

d) normal component of acceleration a_N.

(20) 2. A particle is moving in the plane. Suppose you know that at $t = 2$ seconds the position vector is $\vec{r}(2) = \vec{i} + 2\vec{j}$, the velocity vector is $\vec{v}(2) = 2\vec{i} - \vec{j}$ and the acceleration vector is $\vec{a}(2) = -3\vec{i} + 4\vec{j}$. Using this information only, answer the following questions. Do not attempt to find $\vec{r}$, $\vec{v}$ and $\vec{a}$ as functions of t. Assume distance is measured in feet.

At $t = 2$ seconds

a) at which point is the particle located?

b) what is the speed?

c) what is the unit tangent vector $\vec{T}$?

d) what is the value of a_T?

e) what is the value of a_N?

f) is it speeding up or slowing down? Why?

(10) 3. An object is moving in the plane along the curve $y = 2 - x^2$ from left to right. It is moving at a constant speed of 3 ft/sec.

a) Find a_T and a_N at the point $(x, 2 - x^2)$.

b) Find the velocity vector and the acceleration vector when the object is at the point $(0, 2)$.

(15) 4. As accurately as you can sketch the surfaces in 3- space determined by the following equations

a) $y = x^2$

b) $z = 4 - x^2 - y^2$

c) $z = \sqrt{x^2 + y^2}$

(10) 5. Find the equation of the plane which is tangent to the surface $z = 4 - x^2 - y^2 + 3xy$ at the point $(2, 1, 5)$.

(15) 6. Find all points on the surface $z = y(x^2+2x-3) - \dfrac{y^2}{2}$ at which the tangent plane is horizontal.

(10) 7. Calculate all the second partial derivatives of the function $f(x, y) = e^{xy^2}$.