CALCULUS III NAME _____

EXAM II Rec. Instr. _____

SPRING 2003 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1) Let $f(x, y) = xe^{xy} + 5$.

a) Find the gradient vector field of $f(x, y)$, $\nabla f =$

b) Find the directional derivative of $f(x, y)$ at $(x, y) = (2, 3)$ in the direction of the vector $\vec{a} = \vec{i} - 2\vec{j}$.

c) Find a unit vector which points in the direction which gives the largest directional derivative of $f(x, y)$ at $(x, y) = (2, 3)$.

d) Find the value of the largest directional derivative of $f(x, y)$ at $(x, y) = (2, 3)$.

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(15) 2. Given that $z = f(x, y)$ and $x = 2u - v$, $y = u^2 v$. Suppose at $(x, y) = (0, 16)$ you know that $\frac{\partial f}{\partial x}(0, 16) = 4$ and $\frac{\partial f}{\partial y}(0, 16) = \frac{1}{2}$. Use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ when $(x, y) = (0, 16)$.

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(15) 3) Use the method of **Lagrange Multipliers** to find the largest value and the smallest value of $f(x, y) = x^2 - y$ on the ellipse $4x^2 + y^2 = 36$.

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(25) 4) Find and classify the critical points of

$$f(x, y) = 3x^2y - 2x^3 + y^3 - 12y.$$

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(10) 5) Find the equation of the tangent plane to the surface

$$z^3 + x^3 z + xy^3 + y = 40$$

at $(x, y, z) = (1, 2, 3)$.

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(15) 6) Find the volume of the 3D-region which is enclosed by the surfaces $y = x^2$, $y = 2x$, $z = 1 + 2xy$ and $z = 0$.