CALCULUS III NAME _____

EXAM II Rec. Instr. _____

SPRING 2001 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

**(20)** 1. Find the equation for the tangent plane to the given surface at the specified point.

a) $x^3y + yz^2 + xz = 23$ at $(1, 2, 3)$

b) the graph of $f(x, y) = xe^{xy}$ at $(2, 3, 2e^6)$

**(20)** 2. Given that $z = f(x, y)$ and $x = u - 2v$, $y = u^3 + v^3$. Suppose that at the point $(x, y) = (0, 9)$ you know that $\dfrac{\partial f}{\partial x}(0, 9) = 4$ and $\dfrac{\partial f}{\partial y}(0, 9) = 2$. Use the Chain Rule to calculate $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ when $x = 0$ and $y = 9$.

**(20)** 3. Let $f(x, y) = y\sqrt{x^2 + y^2}$.

a) Find the gradient vector field of $f(x, y)$.

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b) Find the directional derivative of $f(x, y)$ at $(x, y) = (3, 4)$ in the direction of the vector $\vec{a} = -\vec{i} + 2\vec{j}$.

c) Find the largest directional derivative of $f(x, y)$ at $(x, y) = (3, 4)$. In which direction is it obtained?

**(20)** 4. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y) = x^2 + y^2$ on the curve $x^4 + 16y^4 = 16$.

**20)** 5. Find and classify the critical points of $f(x, y) = x^2y - \frac{2}{3}x^3 + 3y - 2y^2$.