CALCULUS III NAME _____

EXAM II Rec. Instr. _____

SPRING 2000 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(10) 1. Given that $z = f(x, y)$ and $x = u^3 - v^3$, $y = u + v$. Suppose you know that at $(x, y) = (0, 2)$, $\dfrac{\partial f}{\partial x}(0, 2) = 4$ and $\dfrac{\partial f}{\partial y}(0, 2) = 2$. Use the chain rule to calculate values for $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ at the point $(x, y) = (0, 2)$.

NAME _____

Rec. Instr. _____

(15) 2. Given $f(x,y) = \dfrac{x}{x^2+y^2}$

a) Find the gradient vector field of $f(x,y)$, $\nabla f =$

b) If you are moving along a straight line through $(x,y) = (1,2)$ toward the point $(x,y) = (3,4)$, find the instantaneous rate of $f(x,y)$ per unit distance that you will observe at $(1,2)$.

c) Find a unit vector which points in the direction you should follow if, starting at $(x,y) = (1,2)$, you want to achieve the most rapid increase possible for $f(x,y)$.

NAME _____

Rec. Instr. _____

(15) 3. Find the equation for the tangent plane to the given surface at the specified point.

a) To the graph of $f(x, y) = y\sqrt{x^2 + 2y}$ at $(x, y, z) = (3, 8, 40)$.

b) To the surface $xz^3 + x^3y + y^3z = 53$ at the point $(x, y, z) = (1, 2, 3)$.

NAME _____

Rec. Instr. _____

(25) 4. Let $f(x,y) = 12xy - x^2y - y^3$. Find and classify the critical points of $f(x,y)$.

NAME _____

Rec. Instr. _____

(15) 5. <u>Use the method of Lagrange multipliers</u> to find the maximum value and the minimum value for $f(x, y) = \frac{1}{3}x^3 - 3x + y^2$ on the circle $x^2 + y^2 = 16$.

NAME _____

Rec. Instr. _____

(10) 6. Evaluate the integral $\int\int_R xy\,dA$ where R is the region in the first quadrant enclosed by the parabolas $4y = x^2$ and $4x = y^2$.

NAME _____

Rec. Instr. _____

(10) 7. First sketch the region of integration corresponding to the iterated integral $\int_0^2 \int_{x^2}^4 \frac{2x}{1+y^2}\, dy\, dx$ and then reverse the order of integration and evaluate the integral.