CALCULUS III NAME _____

EXAM II Rec. Instr. _____

FALL 1999 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. A quantity Q depends upon x and y according to $Q = ye^{-xy}$. Both x and y are changing with time t, and at a certain instant you know that $x = 1$, $y = 2$, $\dfrac{dx}{dt} = 2$ and $\dfrac{dy}{dt} = -2$. Use the chain rule to find $\dfrac{dQ}{dt}$ at this instant.

(10) 2. Find the equation of the tangent plane to the surface $z^3 + xy^3 + x^3y + xz = 7$ at the point $(x, y, z) = (2, 1, -1)$.

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(15) 3. Let $f(x,y) = 2x\sqrt{x+2y}$.

a) Find the gradient vector field of $f(x,y)$, $\nabla f =$

b) Find the directional derivative of $f(x,y)$ at $(x,y) = (1,4)$ in the direction of the vector $\vec{a} = -\vec{i} + 4\vec{j}$.

c) Find a vector which points in the direction which gives the largest directional derivative of $f(x,y)$ at $(x,y) = (1,4)$

d) What is the value of the largest directional derivative of $f(x,y)$ at $(x,y) = (1,4)$?

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(10) 4. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y) = xy$ on the ellipse $x^2 + 3y^2 = 8$.

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(15) 5. Find and classify the critical points of

$$f(x,y) = \frac{1}{2}x^2y - \frac{1}{2}x^2 + \frac{1}{3}y^3 - \frac{3}{2}y^2.$$

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(15) 6. Find the volume of the 3-dimensional region which is under $z = x + xy$ and above the region in the first quadrant of the xy-plane that is bounded by $y = 2x$, $y = 0$, and $x = 2$.

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(10) 7. Evaluate $\int_0^4 \int_{\sqrt{x}}^2 \sqrt{1+y^3}\, dy\, dx$ by <u>first reversing the order of integration</u>.

(10) 8. Find the volume of the 3-dimensional region which is enclosed by the surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$. Use polar coordinates to evaluate the double integral.