CALCULUS III NAME _____

EXAM II Rec. Instr. _____

FALL 1998 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. Find the tangent plane to the given surface at the specified point.

a) To the graph of $f(x, y) = xy$ at the point $(x, y, z) = (2, 3, 6)$.

b) To $x^3y + y^3z + xz^3 = 53$ at $(x, y, z) = (1, 2, 3)$.

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(15) 2. Let $f(x,y) = y\sqrt{x^2+y^2}$.

a) Find the gradient vector field of f.

$\nabla f =$

b) Find the directional derivative of f at $(x,y) = (3,4)$ in the direction of the vector $\vec{v} = 2\vec{i} - \vec{j}$.

c) Find a vector which points in the direction that gives the largest directional derivative for f at $(3,4)$. What is the value of this largest directional derivative?

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(20) 3. Find and classify the critical points of $f(x, y) = 4xy - y^4 - x^2$.

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(15) 4. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y) = x^2 + 2x - y^2$ on the circle $x^2 + y^2 = 16$.

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(10) 5. Use a double integral to calculate the volume of the 3-D region which is under $z = 1 + xy$ and above the region in the first quadrant of the xy-plane which is bounded by $y = 2x$, $x = 0$ and $y = 4$.

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(10) 6. Evaluate $\displaystyle\int_0^2 \int_{y^2}^4 \sqrt{1 + x\sqrt{x}}\, dx\, dy$ by first reversing the order of integration.

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(15) 7. Use a double integral to calculate the volume of the 3-D region which is enclosed by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$. Use polar coordinates to evaluate the double integral.