CALCULUS III NAME _____

EXAM II Rec. Instr. _____

FALL 2002 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1) Find the volume of the 3D-region which is under $z = 1 + x^2$ and above the bounded region in the first quadrant of the xy-plane that is enclosed by the lines $y = x$, $y = 3x$ and $x = 2$.

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(15) 2) <u>Use the method of Lagrange multipliers</u> to find the largest value and the smallest value of $f(x, y) = x + y^2$ on the ellipse $x^2 + 2y^2 = 16$.

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(20) 3) Find and classify the critical points of

$$f(x, y) = 12xy - 3x^2 + y^3 - 15y^2 - 9y.$$

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(15) 4) Given that z is a function of x and y, $z = f(x, y)$, and that $x = s^2 + t$, $y = \dfrac{2s}{t}$ with $s > 0$ and $t > 0$.

Suppose that at $(x, y) = (3, 1)$ we know that $\frac{\partial f}{\partial x}(3, 1) = 2$ and $\frac{\partial f}{\partial y}(3, 1) = 6$. Use the chain rule to find the values of $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $x = 3$ and $y = 1$.

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(20) 5) Let $f(x, y) = 6x\sqrt{x + y^2}$.

a) Find the gradient vector field of f, $\nabla f =$

b) Find the directional derivative of $f(x, y)$ at $(x, y) = (5, 2)$ in the direction of the vector $\vec{v} = 2\vec{i} - 3\vec{j}$.

c) What is the value of the largest directional derivative of $f(x, y)$ at $(5, 2)$? Find a unit vector which points in the direction which gives this largest directional derivative.

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(20) 6) Find the equation of the tangent plane to each of the surfaces at the specified point.

a) to $z = ye^{xy}$ at $(1, 2, 2e^2)$

b) to $xz^3 + zy^3 + yx^3 = 53$ at $(1, 2, 3)$