CALCULUS III NAME _____

EXAM II Rec. Instr. _____

FALL 2000 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

**(15)** 1) Find the equation of the tangent plane to the given surface at the specified point.

a) $x^3 + y^3 + xyz^3 = 41$ at $(3, 2, 1)$

b) $z = 2x\sqrt{x + y^2}$ at $(5, 2, 30)$

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**(20)** 2) Given $f(x, y) = ye^{xy}$

a) Find the gradient vector field of $f(x, y)$.

$\nabla f =$

b) At the point $(x, y) = (-1, 2)$ find the instantaneous rate of change of $f(x, y)$ per unit distance in the direction of the vector $\vec{v} = \vec{i} + 6\vec{j}$.

c) Find a unit vector which points in the direction you should follow, if starting at $(x, y) = (-1, 2)$, you want to achieve the most rapid increase possible for $f(x, y)$.

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**(20)** 3) Let $f(x, y) = 4xy - xy^2 - x^3$. Find and classify the critical points of $f(x, y)$.

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**(10)** 4) Use the method of Lagrange multipliers to find the maximum value and the minimum value of $f(x, y, z) = 2x + 3y + z$ subject to the two constraints $x^2 + y^2 = 20$ and $x + y + z = 0$.

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**(15)** 5) Find the volume of the 3-dimensional region which is under the plane $z = 1 + 2y$ and above the bounded region of the $xy$-plane which is enclosed by $y = x^2$ and $y = 4$.

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**(15)** 6) Use a double integral in polar coordinates to calculate the volume of the 3-dimensional region enclosed by the surfaces $z = x^2 + y^2$ and $z = 6 - \sqrt{x^2 + y^2}$.