CALCULUS III NAME _____

EXAM I Rec. Instr. _____

SPRING 1999 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1. An object is moving in 3-space according to the parametric equations $x = t\,,\ y = t\,,\ z = t^2$. Find as functions of the time t

a) the position vector $\vec{r} =$

b) the velocity vector $\vec{v} =$

c) the acceleration vector $\vec{a} =$

d) the speed $\dfrac{ds}{dt} =$

e) the tangential component of acceleration $a_T =$

f) The curvature $\kappa =$

g) the normal component of acceleration $a_N =$

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(15) 2. An object is moving in the plane in such a way that its acceleration vector as a function of time t is $\vec{a} = (\cos t)\vec{i} + \vec{j}$. Suppose at time $t = 0$, the velocity vector is $\vec{v}(0) = \vec{j}$ and the position vector is $\vec{r}(0) = \vec{i} + \vec{j}$. Find the velocity vector and the position vector as functions of t and then give the parametric equations for the motion.

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(15) 3. An object is moving along a curve in 3-space at a constant speed of 4 ft/sec. Suppose the curve is not a straight line and the curvature as a function of time is $\kappa(t) = \dfrac{1}{1+t^2}$.

a) Explain why the acceleration vector $\vec{a}$ is not $\vec{0}$, even though the speed is constant, and find $|\vec{a}|$ as a function of t.

b) Show that the velocity vector is perpendicular to the acceleration vector at each instant.

Note: This is just like two of the practice problems.

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(20) 4. Given the points $P(1,1,0)$, $Q(2,2,2)$, $R(3,4,0)$, let Δ denote the triangle having P, Q and R as its vertices.

a) Find the angle of Δ at the vertex P.

b) Find the area of the triangle Δ.

c) Find the equation of the plane which contains Δ.

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(15) 5. Find parametric equations for the following lines:

a) The line through $P(2,1,0)$ and $Q(1,3,4)$.

b) The line through $P(1,3,2)$ and perpendicular to the plane $2x-4y+3z=2$.

c) The line of intersection of the planes $x+y+z=0$ and $2x+3y+z=6$.

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(15) 6. Let $\begin{aligned} \vec{a} &= 2\vec{i} - \vec{j} + \vec{k} \\ \vec{b} &= 2\vec{i} + \vec{j} + 2\vec{k} \end{aligned}$

a) $comp_{\vec{b}}\,\vec{a} =$

b) Find vectors $\vec{a}_{\parallel}$ and $\vec{a}_{\perp}$ such that $\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$, $\vec{a}_{\parallel}$ is parallel to $\vec{b}$ and $\vec{a}_{\perp}$ is perpendicular to $\vec{b}$.