CALCULUS III NAME _____

EXAM I Rec. Instr. _____

SPRING 1997 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(10) 1. For the parametric curve $x = t^3 + t$, $y = 1 + t + \sin t$, find $\dfrac{dy}{dx}$ as a function of t. Now find the equation of the line which is tangent to the curve at the point when $t = 0$.

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(10) 2. Calculate the arc length of the curve

$$x = \frac{1}{2}t^2, \quad y = \frac{1}{3}t^3, \quad 1 \le t \le 2.$$

(20) 3. A particle is moving in the plane according to the parametric equations $x = t^2$, $y = t^3 - 3t$ where t is the time. Find, as functions of t,

a) position vector $\vec{r}(t) =$

b) velocity vector $\vec{v}(t) =$

c) acceleration vector $\vec{a}(t) =$

d) speed $\dfrac{ds}{dt} =$

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(20) 4. An object is moving in the plane in such a way that its acceleration vector as a function of time t is $\vec{a} = (\cos t)\,\vec{i} + 2\vec{j}$. Suppose we know that at time $t = 0$ the velocity vector is $\vec{v}(0) = \vec{j}$ and the position vector is $\vec{r}(0) = \vec{i} + \vec{j}$. Find the velocity vector and the position vector as functions of t. Now write the parametric equations of the motion.

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(20) 5. Given the 3-dimensional vectors

$$\begin{aligned} \vec{a} &= 2\vec{i} - \vec{j} + 3\vec{k} \\ \vec{b} &= \vec{i} + \vec{j} - 2\vec{k} \\ \vec{c} &= 3\vec{i} + \vec{j} - 2\vec{k} \end{aligned}$$

Find

a) the cosine of the angle between $\vec{a}$ and $\vec{b}$

b) $\vec{a} \times \vec{b}$

c) the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$

d) the volume of the parallelepiped determined by $\vec{a}$, $\vec{b}$ and $\vec{c}$.

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(10) 6. Find the parametric equations for each of the following lines.

a) line through the points $(1, 3, 2)$ and $(2, 1, 4)$

b) line through $(2, -1, 0)$ and perpendicular to the plane $3x + 4y - 2z = 10$.

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(10) 7. Find the equation for each of the following planes

a) plane containing $(3, 2, -1)$ and perpendicular to the line $x = 1 - t$, $y = 1 + 4t$, $z = 3t$

b) plane containing the points $(1, 0, 0)$, $(2, 1, -1)$, $(1, 2, 1)$