CALCULUS III NAME ______________________

EXAM I Rec. Instr. ______________________

SPRING 2002 Rec. Time ______________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(30) 1) An object is moving in 3-space according to the parametric equations $x = t$, $y = t^2$, $z = 3t^2$. Find as functions of the time t,

a) the position vector; $\vec{r} =$

b) the velocity vector; $\vec{v} =$

c) the acceleration vector; $\vec{a} =$

d) the speed; $\dfrac{ds}{dt} =$

e) the tangential component of acceleration; $a_T =$

f) the curvature; $\kappa =$

g) the normal component of acceleration; $a_N =$

h) the unit tangent vector; $\vec{T} =$

Name: ______________________

Recitation Time: ______________________

(15) 2) An object is moving in 3-space in such a way that its acceleration vector as a function of time t is $\vec{a} = (\cos t)\vec{\imath} - (\sin t)\vec{\jmath} - \vec{k}$. Suppose you know that at time $t = 0$ the velocity vector is $\vec{v}(0) = \vec{\jmath} + \vec{k}$ and the position vector is $\vec{r}(0) = 10\vec{k}$. Find the velocity vector and the position vector as functions of time and then give the parametric equations of the motion.

Name: ______________________________

Recitation Time: ______________________________

(15) 3) An object is moving in the plane around the circle $x^2 + y^2 = 100$. It is moving in such a way that its speed as a function of the time t is

$$\text{speed} = 3t^2.$$

a) Find a_T and a_N as functions of the time t.

b) Suppose the object is moving around the circle in the clockwise direction and at $t = 1$ it is located at the point $(10, 0)$. Find the velocity vector and the acceleration vector when $t = 1$.

Name: ______________________

Recitation Time: ______________________

(20) 4) Given the four points $P(1,1,1)$, $Q(3,2,-1)$, $R(4,1,5)$, $S(1,1,10)$ in 3-space.

a) Find the angle (in degrees) at the vertex P of the triangle having vertices P, Q, and R.

b) Find the area of the triangle having vertices P, Q, and R.

c) Find the volume of the parallelepiped determined by the vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$, and $\overrightarrow{PS}$.

Name: ______________________

Recitation Time: ______________________

(20) 5.a) The force $\vec{F} = 4\vec{\imath} + 2\vec{\jmath} - \vec{k}$ acts on an object as it moves along the straight line from $P(2,1,0)$ to $Q(1,2,3)$. Calculate the work done by $\vec{F}$.

b) Given the vectors $\vec{a} = \vec{\imath} + \vec{\jmath}$, $\vec{b} = 3\vec{\imath} - 2\vec{\jmath}$ and $\vec{c} = -\vec{\imath} + 2\vec{\jmath}$. Find numbers r and s so that $\vec{c} = r\vec{a} + s\vec{b}$.