CALCULUS III NAME _____

EXAM I Rec. Instr. _____

SPRING 2001 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

**(25)** 1. An object is moving in the plane according to the parametric equations $x = \frac{1}{2}t^2$, $y = \frac{1}{3}t^3$ where $t$ is the time.
Find as functions of time

a) position vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) acceleration vector $\vec{a} =$

d) speed $\frac{ds}{dt} =$

e) the tangential component of acceleration $a_T =$

f) curvature $\kappa =$

g) the normal component of acceleration $a_N =$

**(20)** 2. An object is moving in 3-space in such a way that its acceleration vector as a function of time is $\vec{a} = \vec{j} + (\cos\, t)\vec{k}$. Suppose that at time $t = 0$ you know that its velocity vector is $\vec{v}(0) = \vec{i}$ and its position vector is $\vec{r}(0) = \vec{k}$.

a) Find the velocity vector and the position vector as functions of time $t$.

b) Give the parametric equations for the motion.

c) Find the curvature at time $t = 2\pi$.

**(15)** 3. An object is moving in the plane along the curve $y = \frac{1}{2}x^2$, from left to right. It is moving at a constant speed of 3 ft/sec.

a) Find $a_T$ and $a_N$ when the object is at $\left(x, \frac{1}{2}x^2\right)$.

b) Find the velocity vector and the acceleration vector when the object is at the point $\left(1, \frac{1}{2}\right)$.

**(25)** 4. Given the three points $P(1,0,1)$, $Q(2,0,0)$, and $R(-1,2,2)$

a) Find the area of the triangle having $P$, $Q$ and $R$ as vertices.

b) Find the angle at the vertex $P$ of the triangle having $P$, $Q$ and $R$ as vertices.

c) Find the equation of the plane containing the points $P$, $Q$ and $R$.

d) Find the equations for the line which passes through the point $R$ and is perpendicular to the plane of part c).

**(15)** 5. Given the vectors $\vec{a} = \vec{i} + \vec{j} - \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} + 5\vec{k}$, $\vec{c} = \vec{i} + \vec{k}$

a) Calculate the volume of the parallelepiped which is determined by the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ when they are placed with the same initial point.

b) If $\vec{F} = 3\vec{a}$ is a force vector which acts on an object as it moves along the line segment from $(4, 2, 1)$ to $(0, 4, 2)$ calculate the work done by $\vec{F}$. Suppose distance is measured in feet and force in pounds.