CALCULUS III NAME _____

EXAM I Rec. Instr. _____

SPRING 2000 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(15) 1. $x = \sin t - \cos t$, $y = \sin t + \cos t$ is a parametric curve in the plane.

a) Find $\dfrac{dy}{dx}$ as a function of t.

b) Find the equation of the tangent line to the curve when $t = 0$.

c) Find the equation of the tangent line to the curve when $t = \dfrac{\pi}{2}$.

d) Find the arc length of the piece of the curve obtained when $0 \le t \le \dfrac{\pi}{2}$.

NAME _____

Rec. Instr. _____

(20) 2. The three points in space $A(1,0,1)$, $B(4,2,1)$, $C(0,1,2)$ determine a triangle with the points as vertices.

a) Find the angle of the triangle at the vertex A.

b) Calculate the area of the triangle.

c) Find the equation of the plane which contains the triangle.

d) Give parametric equations for the line which passes through the point C and is perpendicular to the triangle.

NAME _____

Rec. Instr. _____

(20) 3. Given the three vectors $\vec{a} = \vec{i} + \vec{j} + 2\vec{k}$, $\vec{b} = 2\vec{i} - 6\vec{k}$, $\vec{c} = \vec{j} + \vec{k}$,

a) Calculate the volume of the parallelepiped determined when $\vec{a}$, $\vec{b}$ and $\vec{c}$ are placed with the same initial point.

b) Find $\text{comp}_{\vec{b}}\, \vec{a}$.

c) Find a vector $\vec{v}$ so that $\vec{v}$ is parallel to $\vec{b}$ and $\vec{a} - \vec{v}$ is perpendicular to $\vec{b}$.

NAME _____

Rec. Instr. _____

(20) 4. An object is moving in the plane according to the parametric equations $x = \cos t$, $y = 3\sin t$ where t is the time.

Find as functions of the time t,

a) position vector $\vec{r} =$

b) velocity vector $\vec{v} =$

c) acceleration vector $\vec{a} =$

d) speed $\dfrac{ds}{dt} =$

e) tangential component of acceleration $a_T =$

f) curvature $\kappa =$

g) sketch the path of motion

h) what is the largest value of the curvature the object encounters? The smallest value?

NAME _____

Rec. Instr. _____

(15) 5. An object is moving in 3-space in such a way that its acceleration vector as a funtion of time is $\vec{a} = \vec{j} + (\sin t)\vec{k}$. Suppose that at time $t = 0$ the velocity vector is $\vec{v}(0) = \vec{i} - \vec{k}$ and the position vector is $\vec{r}(0) = \vec{i} + \vec{j}$.

a) Find the velocity vector as a funtion of t.

b) Find the position vector as a function of t.

c) Write the parametric equations of motion.

d) Find the curvature when $t = \dfrac{\pi}{2}$.

NAME _____

Rec. Instr. _____

(10) 6. A projectile is fired from the top of a 320 foot building. Its angle of inclination with the horizontal is $30°$. If its initial speed is 576 ft/sec find the horizontal distance from its launching point to the point where it hits the earth.