CALCULUS III NAME _____

EXAM I Rec. Instr. _____

FALL 1998 Rec. Time _____

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1. An object is moving in 3-space in such a way that its acceleration vector as a function of time is $\vec{a}(t) = (-\sin t)\vec{j}$. At time $t = 0$ its velocity vector is $\vec{v}(0) = \vec{i} + \vec{j} + \vec{k}$ and its position vector is $\vec{r}(0) = \vec{i}$.

a) Find the velocity vector and the position vector as functions of the time t.

b) Write the parametric equations of the motion.

c) Find a_T the tangential component of acceleration as a function of time.

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(10) 2. An object is moving in the plane around the ellipse $x^2 + 100y^2 = 100$. It is moving at a constant speed of 2 ft/sec.

a) Specify which of the following statements is(are) true and explain why.

i) At each instant the acceleration vector is parallel to the velocity vector.

ii) The acceleration vector is zero.

iii) At each instant the acceleration vector is perpendicular to the velocity vector.

b) Find the points on the ellipse at which a_N take on its largest value and the points at which it takes on its smallest value.

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(15) 3. Given vectors

$$\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$$

$$\vec{b} = \vec{i} + 3\vec{j} + 2\vec{k}$$

$$\vec{c} = 3\vec{i} + \vec{j} - \vec{k}$$

Find (assume the vectors are placed with the same initial point)

a) the cosine of the angle between $\vec{a}$ and $\vec{b}$

b) the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$

c) the volume of the parallelepiped determined by $\vec{a}$, $\vec{b}$ and $\vec{c}$.

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(15) 4. Find parametric equations for each of the following lines:

a) the lines through the points $P_1(2, 1, -3)$, $P_2(1, 4, 3)$.

b) the line through $(1, 2, -1)$ and parallel to the line of intersection of the two planes $2x + y - z = 0$ and $3x - y = 0$.

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(15) 5. Find the equation for each of the following planes:

a) the plane containing the point $(1, 2, -1)$ and perpendicular to the line $x = 2 + 3t,\ y = 1 - t,\ z = 4t$.

b) the plane containing the three points $P(1, 2, 1),\ Q(0, 2, 2),\ (3, 3, 1)$.

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(15) 6. Carefully sketch the following surfaces:

a) $z = x^2$

b) $x^2 + y^2 = 4$

c) $z = 6 - x^2 - y^2$

d) $x^2 + z^2 = y^2$

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(10) 7. Sketch the curve $x = t,\ y = t^2,\ z = t$ and calculate its curvature as a function of t.

Note: in sketching the curve it might help to note that it lies on the surface $y = x^2$.